The Online Appendix for "Quality, Variable Markups, and Welfare: A Quantitative General Equilibrium Analysis of Export Prices"

A Derivation of Demand Function

The utility of a consumer in country j takes the following form:

$$U_j = \left\{ \sum_i \int_{\omega\in\Omega_{ij}} \left[\left(q_{ij}(\omega) x^c_{ij}(\omega) + \bar{x} \right)^{\frac{\sigma-1}{\sigma}} - \bar{x}^{\frac{\sigma-1}{\sigma}} \right] d\omega \right\}^{\frac{\sigma}{\sigma-1}} \tag{A.1}$$

subject to the following budget constraint:

$$\sum_i \int_{\omega\in\Omega_{ij}} p_{ij}(\omega) x^c_{ij}(\omega) d\omega \leq y_j \tag{A.2}$$

So that the Lagrange function can be written as: $L = \left\{ \sum_i \int_{\omega\in\Omega_{ij}} \left[\left(q_{ij}(\omega) x^c_{ij}(\omega) + \bar{x} \right)^{\frac{\sigma-1}{\sigma}} - \bar{x}^{\frac{\sigma-1}{\sigma}} \right] d\omega \right\}^{\frac{\sigma}{\sigma-1}} + \lambda \left(y_j - \sum_i \int_{\omega\in\Omega_{ij}} p_{ij}(\omega) x^c_{ij}(\omega) d\omega \right)$,where λ is the Lagrange multiplier, y_j denotes the consumer's income. Taking the first order condition with respect to $x^c_{ij}(\omega)$ yields:

$$\lambda \tilde{p}_{ij}(\omega) = U_j^{\frac{1}{\sigma}} \left(q_{ij}(\omega) x^c_{ij}(\omega) + \bar{x} \right)^{-\frac{1}{\sigma}}, \tag{A.3}$$

where $\tilde{p}_{ij}(\omega) = p_{ij}(\omega) / q_{ij}(\omega)$ is the quality adjusted price. Following Jung, Simonovska and Weinberger (2019), we define $P_{j\sigma} = \left\{ \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega)^{1-\sigma} d\omega \right\}^{\frac{1}{1-\sigma}}$, and $P_j = \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega$. The budget constraint can be rewritten as:

$$\begin{aligned} y_j + \bar{x} P_j &= \sum_i \int_{\omega\in\Omega_{ij}} \tilde{p}_{ij}(\omega) \left(q_{ij}(\omega) x^c_{ij}(\omega) + \bar{x} \right) d\omega \\ &= \frac{U_j}{\lambda^\sigma} \sum_i \int_{\omega\in\Omega_{ij}} \left(\tilde{p}_{ij}(\omega) \right)^{1-\sigma} d\omega = \frac{U_j}{\lambda^\sigma} P^{1-\sigma}_{j\sigma} \end{aligned} \tag{A.4}$$

where the second equility stems from equation (A.3). The previous equation (A.4) could be rewritten as $\frac{U_j}{\lambda^\sigma} = \frac{y_j + \bar{x} P_j}{P^{1-\sigma}_{j\sigma}}$, which, together with equation (A.3), implies:

$$x_{ij}(\omega) = x^c_{ij}(\omega) L_j = \frac{L_j}{q_{ij}(\omega)} \left[\frac{U_j}{\lambda^\sigma \left(\tilde{p}_{ij}(\omega) \right)^\sigma} - \bar{x} \right] = \frac{L_j}{q_{ij}(\omega)} \left[\frac{y_j + \bar{x} P_j}{P^{1-\sigma}_{j\sigma}} \left(\frac{p_{ij}(\omega)}{q_{ij}(\omega)} \right)^{-\sigma} - \bar{x} \right] \tag{A.5}$$

B Log Utility Function

The utility of a consumer in country j takes the log utility function form:

$$U_j = \sum_i \int_{\omega \in \Omega_{ij}} \left[\log \left(q_{ij}(\omega) x_{ij}^c(\omega) + \bar{x} \right) - \log \bar{x} \right] d\omega \tag{B.1}$$

Based on the same derivation as in Appendix (A), the representative consumer in country j's demand satisfies:

$$x_{ij}(\omega) = x_{ij}^c(\omega) L_j = \frac{\bar{x} L_j}{q_{ij}(\omega)} \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1 \right] \tag{B.2}$$

where $\tilde{p}_{ijs}(\omega) = \frac{p_{ij}(\omega)}{q_{ij}(\omega)}$ and $\psi_j = \frac{y_j + \bar{x} P_j}{\bar{x} N_j}$. The aggregate prices satisfies $P_j = \sum_i \int_{\omega \in \Omega_{ij}} \tilde{p}_{ij}(\omega) d\omega$. Now, sales and profit for a given variety exported from i to j are as follows,

$$r_{ij}(\omega) = \bar{x} L_j \tilde{p}_{ij}(\omega) \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1 \right] \tag{B.3}$$

$$\pi_{ij}(\omega) = \bar{x} L_j \left[\tilde{p}_{ij}(\omega) - \tilde{c}_{ij}(\omega) \right] \left[\frac{\psi_j}{\tilde{p}_{ij}(\omega)} - 1 \right] \tag{B.4}$$

where $\tilde{c}_{ij}(\omega) = \frac{c_{ij}(\omega)}{q_{ij}(\omega)}$ is the quality-adjusted marginal cost. Given the quality adjusted marginal cost, firms maximize their profits. This implies that the optimal quality adjusted price of the good satisfies:

$$\tilde{p}_{ij}(\omega) = \sqrt{\psi_j \tilde{c}_{ij}(\omega)}$$

We assume that the marginal cost of producing a variety of final good with quality q_{ij} by a firm with productivity φ is given by:

$$c_{ij}(\varphi, \varepsilon) = \left(T_{ij} w_i + \frac{w_i \tau_{ij}}{\varphi} q_{ij}^{\eta} \right) \varepsilon$$

where τ_{ij} is ad valorem trade cost and T_{ij} is a specific transportation cost from country i to country j. Maximizing the profit is equivalent to minimizing the quality-adjusted cost $\tilde{c}_{ij}(\omega)$ by the envelop theorem. Choosing the quality to minimize the quality-adjusted marginal cost implies that the optimal level of quality for a firm with productivity φ is:

$$q_{ij}(\varphi, \varepsilon) = \left(\frac{T_{ij} \varphi}{(\eta - 1) \tau_{ij}} \right)^{\frac{1}{\eta}} \tag{B.5}$$

and hence the quality adjusted marginal cost of production now is:

$$\tilde{c}_{ij}(\varphi, \varepsilon) = \left(\frac{\eta}{\eta - 1} T_{ij} w_i \right)^{\frac{\eta - 1}{\eta}} \left(\frac{\varphi}{\eta w_i \tau_{ij}} \right)^{-\frac{1}{\eta}} \varepsilon \tag{B.6}$$

At the productivity cutoff $\varphi_{ij}^*(\varepsilon)$, we have $\tilde{p}_{ij}^*(\varphi, \varepsilon) = \tilde{c}_{ij}^*(\varphi, \varepsilon) = \psi_j$, which implies that the productivity cutoff $\varphi_{ij}^{*}(\varepsilon)$ takes the following form:

$$\varphi_{ij}^{*}(\varepsilon)=\varphi_{ij}^{*}\varepsilon^{\eta}=\frac{\eta^{\eta}}{(\eta-1)^{\eta-1}}T_{ij}^{\eta-1}\tau_{ij}w_{i}^{\eta}(\psi_{j})^{-\eta}\varepsilon^{\eta},$$

In the log utility function, price could be written as:

$$p_{ij}(\varphi,\varepsilon)=\tilde{p}_{ij}(\varphi,\varepsilon)q_{ij}(\varphi,\varepsilon)=\left[\frac{\varphi}{\varphi_{ij}^{*}(\varepsilon)}\right]^{\frac{1}{2\eta}}\frac{\eta}{\eta-1}T_{ij}w_{i}\varepsilon.$$

Different from the CES utility function, now the markup function could be expressed explicitly as $\left[\frac{\varphi}{\varphi_{ij}^{*}(\varepsilon)}\right]^{\frac{1}{2\eta}}$.

C Derivation for P_j, $P_{j\sigma}$, X_{ij} and π_i

To derive the aggregate variables, we define $t_{ij}=\tilde{p}_{ij}(\omega)/p_j^{*}$. Following the insight of Arkolakis et al. (2019) and Jung, Simonovska and Weinberger (2019), this will make the integration not country specific. From equations (9) and (11), we have:

$$\frac{\tilde{c}_{ij}(\varphi,\varepsilon)}{\tilde{p}_{j}^{*}}=\frac{\tilde{c}_{ij}(\varphi,\varepsilon)}{\tilde{c}_{ij}^{*}(\varphi,\varepsilon)}=\left(\frac{\varphi}{\varphi_{ij}^{*}(\varepsilon)}\right)^{-\frac{1}{\eta}} \tag{C.1}$$

Combining the above equation with equation (6) we have:

$$\sigma\left(\frac{\varphi}{\varphi_{ij}^{*}(\varepsilon)}\right)^{-\frac{1}{\eta}}=t_{ij}^{\sigma+1}+(\sigma-1)\,t_{ij} \tag{C.2}$$

which implies that t_{ij} is a monotonically decreasing function of φ. Note that t_{ij} will lies between $(0,1]$ since $\varphi\in\left[\varphi_{ij}^{*}(\varepsilon),\infty\right)$. Totally differentiating both sides gives us:

$$d\varphi=-\eta\sigma^{\eta}\varphi_{ij}^{*}(\varepsilon)\frac{(\sigma+1)\,t_{ij}^{\sigma}+(\sigma-1)}{\left[t_{ij}^{\sigma+1}+(\sigma-1)\,t_{ij}\right]^{1+\eta}}dt_{ij} \tag{C.3}$$

First, we derive $P_{j\sigma}$. By definition, we have:

$$\begin{aligned}P_{j\sigma}&=\left\{\sum_{i}N_{ij}\int_{0}^{\infty}\int_{\varphi_{ij}^{*}(\varepsilon)}^{\infty}\tilde{p}_{ij}(\varphi,\varepsilon)^{1-\sigma}\mu_{ij}(\varphi,\varepsilon)\,f(\varepsilon)\,d\varphi d\varepsilon\right\}^{\frac{1}{1-\sigma}}\\&=\tilde{p}_{j}^{*}\left\{\sum_{i}N_{ij}\int_{0}^{\infty}\left[\int_{\varphi_{ij}^{*}(\varepsilon)}^{\infty}t_{ij}^{1-\sigma}\mu_{ij}(\varphi,\varepsilon)\,d\varphi\right]f(\varepsilon)\,d\varepsilon\right\}^{\frac{1}{1-\sigma}}\end{aligned} \tag{C.4}$$

Plugging in the expression of conditional density $\mu_{ij}(\varphi,\varepsilon)$ into equation (C.4) and then we transform the integration variable from φ to t_{ij} by using the relationship between φ and t_{ij}, the inner integration with respect to productivity can be written as:

$$\int_{\varphi_{ij}^*(\varepsilon)}^{\infty} t_{ij}^{1-\sigma}\mu_{ij}(\varphi,\varepsilon)\,d\varphi = \frac{\eta\theta}{\sigma^{\eta\theta}}\int_0^1 t_{ij}^{1-\sigma}\left[t_{ij}^{\sigma+1}+(\sigma-1)\,t_{ij}\right]^{\eta\theta-1}\left[(\sigma+1)\,t_{ij}^{\sigma}+(\sigma-1)\right]dt_{ij}$$

which is a constant, and we denote it as β_σ. Thus,

$$P_{j\sigma} = \beta_\sigma^{\frac{1}{1-\sigma}}\tilde{p}_j^* N_j^{\frac{1}{1-\sigma}}$$

Second, we derive P_j. By definition, we have

$$\begin{aligned}
P_j &= \sum_i N_{ij}\int_0^\infty\int_{\varphi_{ij}^*(\varepsilon)}^{\infty}\tilde{p}_{ij}(\varphi,\varepsilon)\,\mu_{ij}(\varphi,\varepsilon)\,f(\varepsilon)\,d\varphi d\varepsilon\\
&= \tilde{p}_j^*\sum_i N_{ij}\int_0^\infty\left[\int_{\varphi_{ij}^*(\varepsilon)}^{\infty} t_{ij}\mu_{ij}(\varphi,\varepsilon)\,d\varphi\right]f(\varepsilon)\,d\varepsilon\\
&= \beta\tilde{p}_j^* N_j
\end{aligned}$$

In the last equality, we use the same variable transformation method as before where β is a constant, defined by:

$$\beta = \frac{\eta\theta}{\sigma^{\eta\theta}}\int_0^1 t_{ij}\left[t_{ij}^{\sigma+1}+(\sigma-1)\,t_{ij}\right]^{\eta\theta-1}\left[(\sigma+1)\,t_{ij}^{\sigma}+(\sigma-1)\right]dt_{ij}$$

To derive the equations (C.5) and (C.6), we plug in $\tilde{p}_j^* = \left(\frac{w_j+\bar{x}P_j}{\bar{x}P_{j\sigma}^{1-\sigma}}\right)^{\frac{1}{\sigma}}$ into $P_{j\sigma}$ and P_j, we have:

$$\begin{aligned}
P_{j\sigma} &= \beta_\sigma^{\frac{1}{1-\sigma}}\left(\frac{w_j+\bar{x}P_j}{\bar{x}P_{j\sigma}^{1-\sigma}}\right)^{\frac{1}{\sigma}} N_j^{\frac{1}{1-\sigma}}\\
P_j &= \beta\left(\frac{w_j+\bar{x}P_j}{\bar{x}P_{j\sigma}^{1-\sigma}}\right)^{\frac{1}{\sigma}} N_j,
\end{aligned}$$

which provide us with 2 equations to solve for $P_{j\sigma}$ and P_j. Solving the system yields:

$$\bar{x}P_j = \frac{\beta}{\beta_\sigma-\beta}w_j \tag{C.5}$$

$$\bar{x}P_{j\sigma} = \frac{\beta_\sigma^{\frac{1}{1-\sigma}}}{\beta_\sigma-\beta}N_j^{\frac{\sigma}{1-\sigma}}w_j \tag{C.6}$$

Next, we derive bilateral trade flow X_{ij}, which is given by:

$$\begin{aligned} X_{ij} &= N_{ij} \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty r_{ij}(\varphi,\varepsilon)\, \mu_{ij}(\varphi,\varepsilon)\, d\varphi \right] f(\varepsilon)\, d\varepsilon \\ &= N_{ij}\left(\bar{x}\tilde{p}_j^* L_j\right) \int_0^\infty \left[\int_{\varphi_{ij}^*(\varepsilon)}^\infty t_{ij}\left(t_{ij}^{-\sigma}-1\right) \mu_{ij}(\varphi,\varepsilon)\, d\varphi \right] f(\varepsilon)\, d\varepsilon \\ &= (\beta_\sigma - \beta)\, \bar{x}\tilde{p}_j^* L_j N_{ij} = X_j \frac{N_{ij}}{N_j} \end{aligned}$$

where $X_j = \sum_i X_{ij}$ is total absorption.

Finally, we derive firm's expected average profit π_i, which satisfies:

$$\begin{aligned} \pi_i &= \frac{1}{J_i} \sum_j N_{ij} \int_0^\infty \int_{\varphi_{ij}^*(\varepsilon)}^\infty \pi_{ij}(\varphi,\varepsilon)\, \mu_{ij}(\varphi)\, f(\varepsilon)\, d\varphi d\varepsilon \\ &= \frac{1}{J_i} \beta_\pi \sum_j \bar{x}\tilde{p}_j^* L_j N_{ij} = \frac{1}{J_i} \frac{\beta_\pi}{\beta_\sigma - \beta} \sum_j X_{ij} \\ &= \frac{1}{J_i} \frac{\beta_\pi}{\beta_\sigma - \beta} \sum_j \frac{N_{ij}}{N_j} X_j \end{aligned}$$

where

$$\beta_\pi = \frac{\eta\theta}{\sigma^{\eta\theta}} \int_0^1 \frac{\left(t_{ij}^{\sigma+1} - t_{ij}\right)\left(t_{ij}^{-\sigma}-1\right)}{\sigma} \left[t_{ij}^{\sigma+1} + (\sigma-1)\, t_{ij}\right]^{\eta\theta-1} \left[(\sigma+1)\, t_{ij}^{\sigma} + (\sigma-1)\right] dt_{ij}$$

D Derivation of Welfare Formula

In the following, we proceed to derive the welfare formula in second steps.

Step 1: Extensive Margin is zero

The expenditure function in country j takes the following form:

$$e_j = \min_{x_{ij}^c} \sum_i J_i \int_{\varphi_{ij}^*}^\infty p_{ij}(\varphi)\, x_{ij}^c(\varphi)\, g_i(\varphi)\, d\varphi \tag{D.1}$$

$$\text{s.t.} \quad \left[\sum_i J_i \int_{\varphi_{ij}^*}^\infty \left[\left(q_{ij}(\varphi)\, x_{ij}^c(\varphi) + \overline{x}\right)^{\frac{\sigma-1}{\sigma}} - \overline{x}^{\frac{\sigma-1}{\sigma}} \right] g_i(\varphi)\, d\omega \right]^{\frac{\sigma}{\sigma-1}} \geq U_j \tag{D.2}$$

The Lagrange function can be written as:

$$e_j = \sum_i J_i \int_{\varphi_{ij}^*}^\infty p_{ij}(\varphi)\, x_{ij}^c(\varphi)\, g_i(\varphi)\, d\varphi + \xi \left(U_j - \left[\sum_i J_i \int_{\varphi_{ij}^*}^\infty u\left(q_{ij}(\varphi)\, x_{ij}^c(\varphi)\right) g_i(\varphi)\, d\varphi \right]^{\frac{\sigma}{\sigma-1}} \right) \tag{D.3}$$

where $u\left(q_{ij}(\varphi)\, x_{ij}^c(\varphi)\right) = \left(q_{ij}(\varphi)\, x_{ij}^c(\varphi) + \overline{x}\right)^{\frac{\sigma-1}{\sigma}} - \overline{x}^{\frac{\sigma-1}{\sigma}}$ and ξ is the Lagrange multiplier.

Taking the first order condition with respect to $x_{ij}^c(\omega)$ yields:

$$p_{ij}(\varphi) = \xi U_j^{\frac{1}{\sigma}} \left(q_{ij}(\varphi) x_{ij}^c(\varphi) + \overline{x}\right)^{-\frac{1}{\sigma}} q_{ij}(\varphi), \tag{D.4}$$

By total differentiating the expenditure function e_j, we have:

$$\begin{aligned}
d\ln e_j &= \underbrace{\sum_i J_i \int_{\varphi_{ij}^*}^{\infty} \frac{p_{ij}(\varphi) x_{ij}^c(\varphi) g_i(\varphi)}{e_j} d\ln p_{ij}(\varphi)\, d\varphi}_{\text{Price Effect}} \\
&\quad \underbrace{+\sum_i \frac{J_i g_i\left(\varphi_{ij}^*\right)\left[p_{ij}\left(\varphi_{ij}^*\right) x_{ij}^c\left(\varphi_{ij}^*\right) - \frac{\sigma}{\sigma-1}\xi U_j^{\frac{1}{\sigma}} u\left(\varphi_{ij}^*\right)\right] d\varphi_{ij}^*}{e_j}}_{\text{Extensive Margin from Productivity Cutoff}} \\
&\quad \underbrace{+\sum_i \frac{\int_{\varphi_{ij}^*}^{\infty} p_{ij}(\varphi) x_{ij}^c(\varphi) g_i(\varphi)\, d\varphi - \frac{\sigma}{\sigma-1}\xi U_j^{\frac{1}{\sigma}} \int_{\varphi_{ij}^*}^{\infty} u\left(q_{ij}(\varphi) x_{ij}^c(\varphi)\right) g_i(\varphi)\, d\varphi}{e_j} dJ_i}_{\text{Extensive Margin from Potential Firm Mass}} \\
&\quad \underbrace{-\sum_i \frac{\xi U_j^{\frac{1}{\sigma}} J_i \int_{\varphi_{ij}^*}^{\infty} \left(q_{ij}(\varphi) x_{ij}^c(\varphi) + \overline{x}\right)^{-\frac{1}{\sigma}} q_{ij}(\varphi) x_{ij}^c(\varphi) g_i(\varphi)}{e_j} d\ln q_{ij}(\varphi)\, d\varphi}_{\text{Quality Effect}} \\
&= \sum_i J_i \int_{\varphi_{ij}^*}^{\infty} \frac{p_{ij}(\varphi) x_{ij}^c(\varphi) g_i(\varphi)}{e_j} \left(d\ln p_{ij}(\varphi) - d\ln q_{ij}(\varphi)\right) d\varphi
\end{aligned}$$

where the second term "Extensive Margin from Productivity Cutoff" equals zero since $x_{ij}^c\left(\varphi_{ij}^*\right) = 0$ and the third term "Extensive Margin from Potential Firm Mass" also equals zero since the potential firm mass J_i is constant. The second equality stems from equation (D.4).

Step 2: Proof of $d\ln e_j = \left(1 - \frac{\rho}{1+\eta\theta}\right) \frac{d\ln \lambda_{jj}}{\eta\theta}$

Based on equations (11), (13) and (21), we can rewrite N_{ij} as:

$$N_{ij} = \frac{\kappa\beta_\pi}{f\beta_X} b_i L_i \left[\frac{\eta^\eta}{(\eta-1)^{\eta-1}} T_{ij}^{\eta-1} \tau_{ij} w_i^\eta \left(\tilde{p}_j^*\right)^{-\eta}\right]^{-\theta} \tag{D.5}$$

where $\beta_X = \beta_\sigma - \beta$ is a constant. This implies that

$$\lambda_{jj} = \frac{X_{jj}}{\sum_i X_{ij}} = \frac{N_{jj}}{\sum_i N_{ij}} = \frac{b_j L_j \left(T_{jj}^{\eta-1} \tau_{jj} w_j^\eta\right)^{-\theta}}{\sum_i b_i L_i \left(T_{ij}^{\eta-1} \tau_{ij} w_i^\eta\right)^{-\theta}} \tag{D.6}$$

Without loss of generality, we use labor in country j as our numeraire so that $w_j = 1$ before and after the change in trade costs. Consider the foreign shocks: (T_{ij}, τ_{ij}) is changed to (T'_{ij}, τ'_{ij})

for $i \neq j$ such that $T_{jj} = T'_{jj}, \tau_{jj} = \tau'_{jj}$. Totally differentiating the previous equation implies:

$$d\ln\lambda_{jj} = \sum_i \lambda_{ij}\Lambda_{ij} \tag{D.7}$$

where $\Lambda_{ij} = \theta\eta d\ln w_i + \theta\eta d\ln T_{ij} + \theta\left(d\ln\tau_{ij} - d\ln T_{ij}\right)$

The expression of $\tilde{p}_j^*$, together with equation (C.5) and (C.6), imply that:

$$\begin{aligned} d\ln\tilde{p}_j^* &= \frac{\sigma-1}{\sigma} d\ln P_{j\sigma} = -\sum_i \lambda_{ij} d\ln N_{ij} \\ &= \frac{1}{1+\eta\theta}\sum_i \lambda_{ij}\Lambda_{ij} \end{aligned} \tag{D.8}$$

We define $\lambda_{ij} = \int_{\varphi_{ij}^*}^{\infty} \lambda_{ij}(\varphi)\,d\varphi$ to denote the total share of expenditure on goods from country i in country j and define $\lambda_{ij}(\varphi) = \frac{J_i p_{ij}(\varphi) x_{ij}(\varphi) g_i(\varphi)}{\sum_i J_i \int_{\varphi_{ij}^*}^{\infty} p_{ij}(\varphi) x_{ij}(\varphi) g_i(\varphi) d\varphi}$ to denote the share of expenditure in country j on goods produced by firms from country i with productivity φ. According to the equations (9), (12) and (D.8), the percentage change in expenditure satisfies:

$$\begin{aligned} d\ln e_j &= \sum_i \int_{\varphi_{ij}^*}^{\infty} \lambda_{ij}(\varphi)\left(d\ln\widetilde{p}_{ij}(\varphi)\right) d\varphi \\ &= \sum_i \int_{\varphi_{ij}^*}^{\infty} \lambda_{ij}(\varphi)\left(d\ln\widetilde{c}_{ij}(\varphi) + d\ln\mu(\varphi)\right) d\varphi \\ &= \sum_i \lambda_{ij}\left(\frac{\Lambda_{ij}}{\theta\eta} - \rho d\ln\left(\varphi_{ij}^*\right)^{\frac{1}{\eta}}\right) \\ &= \sum_i \lambda_{ij}\left(\frac{\Lambda_{ij}}{\theta\eta} - \rho\left(\frac{\Lambda_{ij}}{\eta\theta} - d\ln\tilde{p}_j^*\right)\right) \\ &= \sum_i \lambda_{ij}\left(\frac{\Lambda_{ij}}{\theta\eta} - \rho\left(\frac{\Lambda_{ij}}{\eta\theta} - \frac{1}{1+\eta\theta}\sum_i \lambda_{ij}\Lambda_{ij}\right)\right) \\ &= \left(1 - \frac{\rho}{1+\eta\theta}\right)\frac{1}{\eta\theta}\sum_i \lambda_{ij}\Lambda_{ij} \\ &= \left(1 - \frac{\rho}{1+\eta\theta}\right)\frac{1}{\eta\theta} d\ln\lambda_{jj} \end{aligned}$$

where $\mu = \frac{\widetilde{p}_{ij}(\varphi)}{\widetilde{c}_{ij}(\varphi)}$ and the third equality is the same as Arkolakis et al. (2019). The markup elasticity $\rho = \int_{\varphi_{ij}^*}^{\infty} \frac{\lambda_{ij}(\varphi)}{\lambda_{ij}} \frac{d\ln\mu(v)}{d\ln v} d\varphi$, where $v = \left(\frac{\varphi}{\varphi_{ij}^*}\right)^{\frac{1}{\eta}}$, satisfies:

$$\rho = \int_{\varphi_{ij}^*}^{\infty} \frac{\frac{\widetilde{p}_{ij}(\varphi)}{\widetilde{p}_j^*}\left[\left(\frac{\widetilde{p}_{ij}(\varphi)}{\widetilde{p}_j^*}\right)^{-\sigma}-1\right] g_i\left(\frac{\varphi}{\varphi_{ij}^*}\right)}{\int_{\varphi_{ij}^*}^{\infty} \frac{\widetilde{p}_{ij}(\varphi)}{\widetilde{p}_j^*}\left[\left(\frac{\widetilde{p}_{ij}(\varphi)}{\widetilde{p}_j^*}\right)^{-\sigma}-1\right] g_i\left(\frac{\varphi}{\varphi_{ij}^*}\right) d\frac{\varphi}{\varphi_{ij}^*}} \frac{d\ln\mu(v)}{d\ln v} d\frac{\varphi}{\varphi_{ij}^*}$$

$$= \int_1^{\infty} \frac{\mu v^{-1}\left[\left(\mu v^{-1}\right)^{-\sigma}-1\right] v^{-\theta\eta-1}}{\int_1^{\infty} \mu v^{-1}\left[\left(\mu v^{-1}\right)^{-\sigma}-1\right] v^{-\theta\eta-1} dv} \frac{d\ln\mu(v)}{d\ln v} dv$$

μ is determined by $\sigma v^{-1} = \left(\mu v^{-1}\right)^{\sigma+1} + (\sigma-1)\mu v^{-1}$.

Consequently, the welfare gains associated with a small trade shock equals to $-\left(1-\frac{\rho}{1+\eta\theta}\right)\frac{d\ln\lambda_{jj}}{\eta\theta}$. Here, we consider a generalized CES function with $\overline{x} > 0$. If we assume that the utility function is CES function (i.e., $\overline{x} = 0$), the markup is constant and $\rho = 0$. Now, the welfare gains associated with a small trade shock become $-\frac{d\ln\lambda_{jj}}{\eta\theta}$.

If the model contains only variable markup but no endogenous quality and no Washington Apples mechanism, our model would be essentially identical to Jung, Simonovska and Weinberger (2019). Now, the welfare gains associated with a small trade shock become $-\left(1-\frac{\rho}{1+\theta}\right)\frac{d\ln\lambda_{jj}}{\theta}$, where $\rho = \int_1^{\infty} \frac{\mu v^{-1}\left[\left(\mu v^{-1}\right)^{-\sigma}-1\right] v^{-\theta-1}}{\int_1^{\infty} \mu v^{-1}\left[\left(\mu v^{-1}\right)^{-\sigma}-1\right] v^{-\theta-1} dv} \frac{d\ln\mu(v)}{d\ln v} dv$ and μ is determined by $\sigma v^{-1} = \left(\mu v^{-1}\right)^{\sigma+1} + (\sigma-1)\mu v^{-1}$.

E Supplementary Figure

Figure A.1: Sales and Markup Distribution

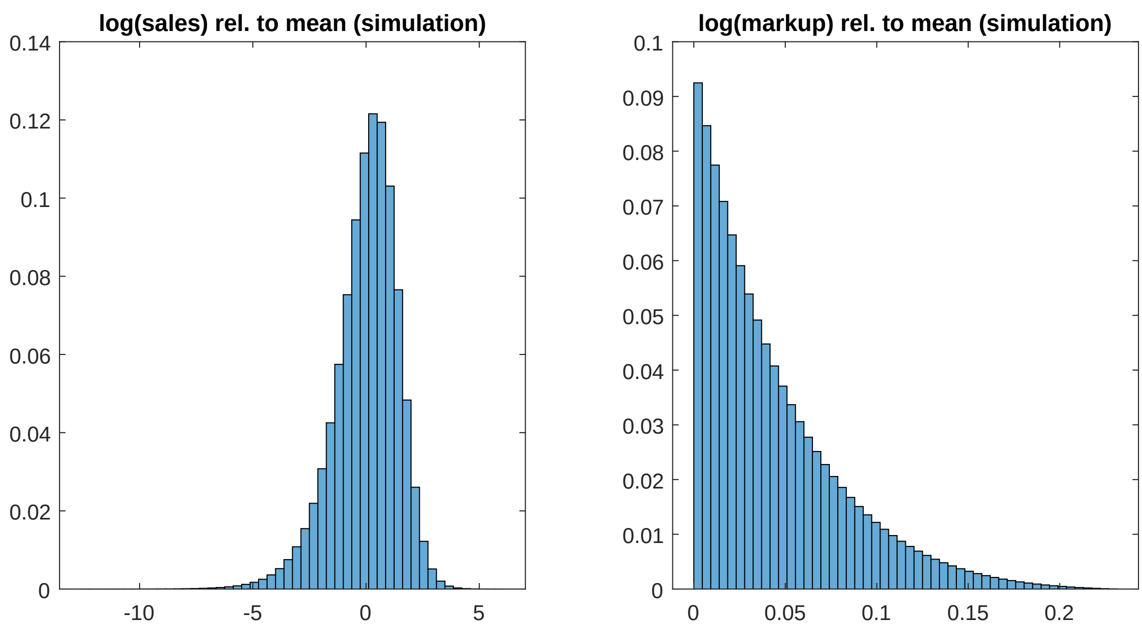


Figure A.2: The relationship between market size and firm-level variables (prices, sales, and quality)

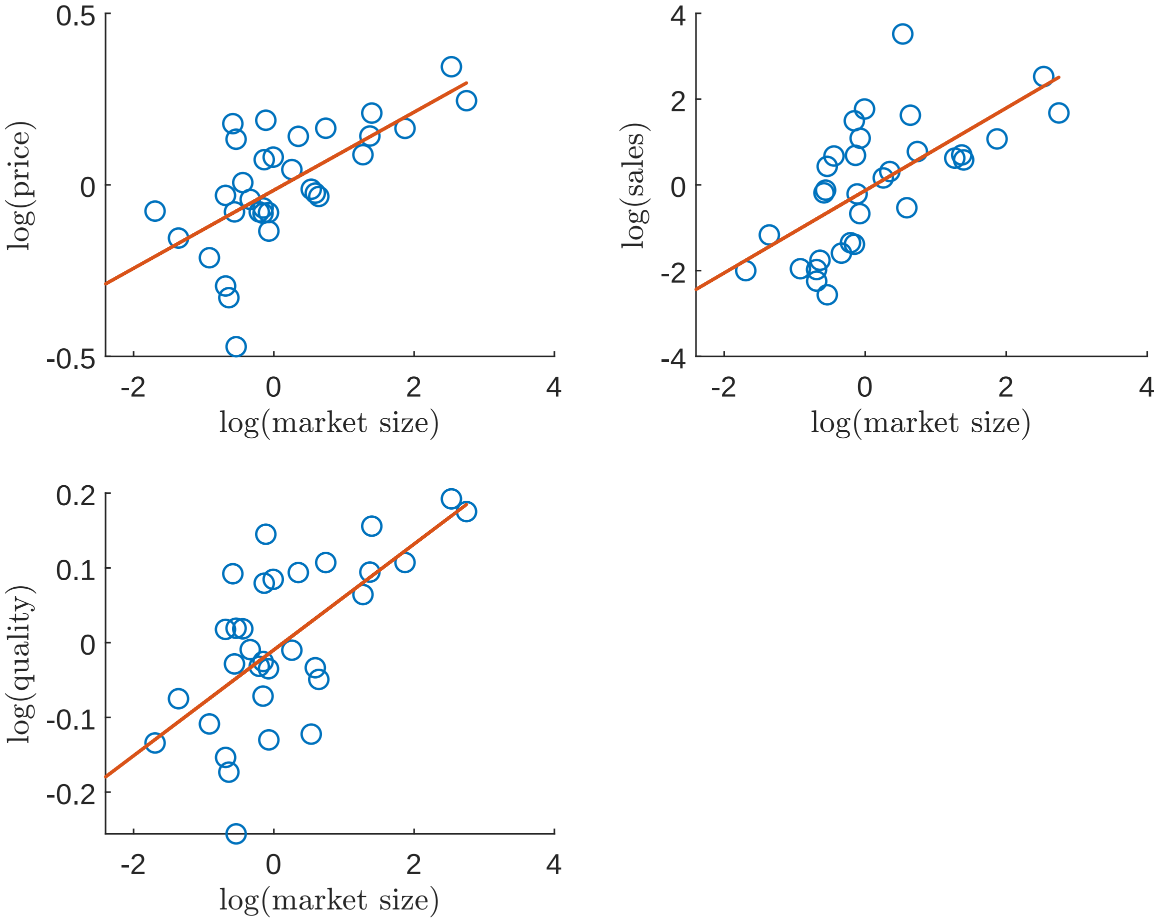
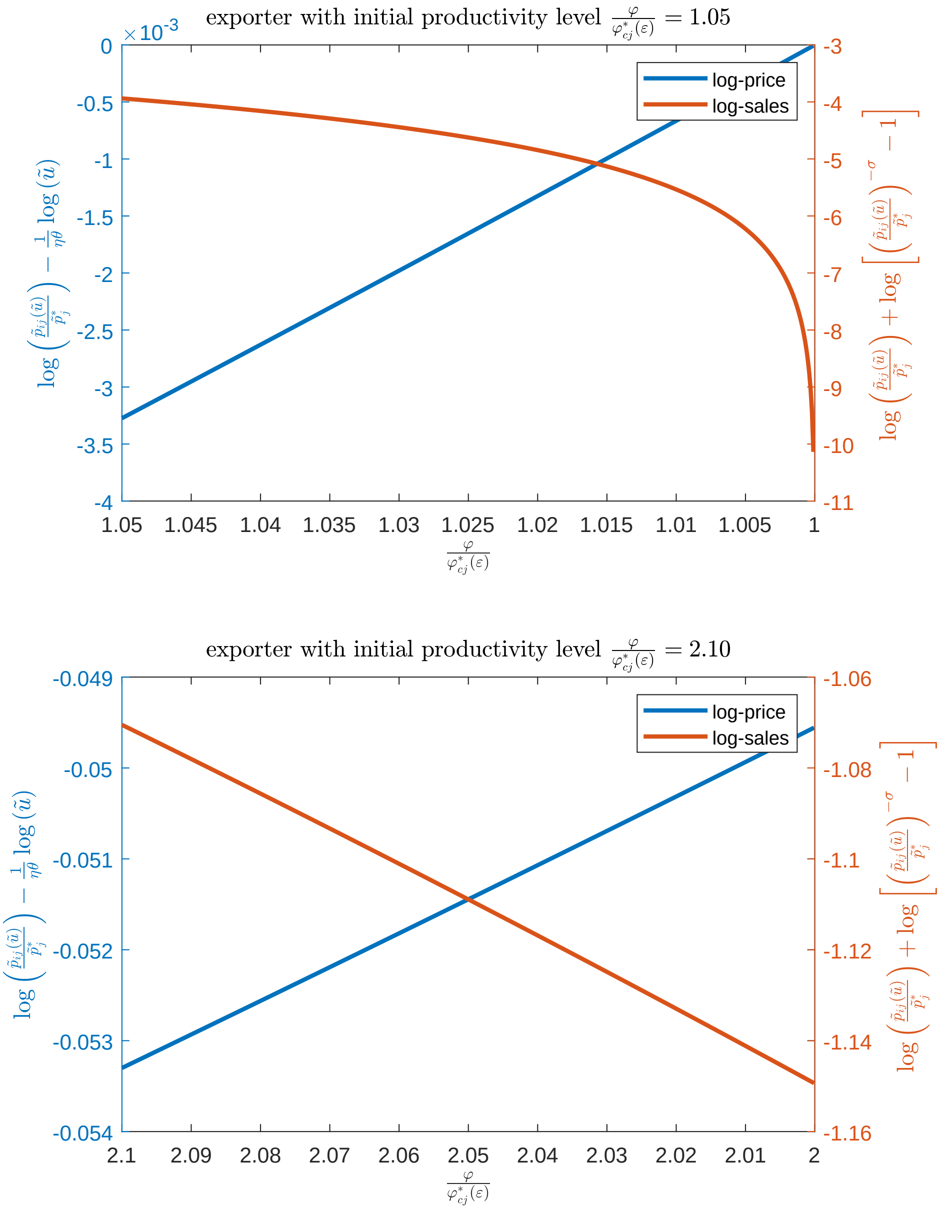


Figure A.3: Illustration: the Changes in Prices and Sales by Low- vs. High-productivity Firms after Trade Cost Shock



Explanatory notes for Figure A.3:

The upper panel plots a low-productivity firm whose productivity is only 5% above the cutoff productivity before the trade shock, i.e., $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)} = 1.05$. When trade cost increases by 5% (either from τ or T), $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)}$ goes to 1. Then, this producer starts to become a marginal exporter. The left y-axis plots the change of log(price), and the right y-axis plots the change of log(sales). Clearly, the variation in price changes is very small whereas the change in sales is large. Next, we turn to a initially high-productivity firm with $\frac{\varphi}{\varphi^*_{cj}(\varepsilon)} = 2.10$ shown in the lower panel. When it is hit by 5% increase in trade cost, the changes in log(price) is similar comparing with the low-productivity exporter in the upper panel, but the change in log(sales) is much smaller for this high-productivity firm.