Firm-to-Firm Trade:

Imports, Exports, and the Labor Market[1]

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Abstract

Customs data and firm-level production data reveal both the heterogeneity and the granularity of individual buyers, and sellers. We seek to capture these firm-level features in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

1 Introduction

International economists have begun to exploit data generated by customs records, which describe the finest unit of trade transactions. These records expose the activity of individual buyers and sellers that underlie the aggregate trade flows, which had been the object of earlier quantitative analysis in international trade.

Some striking regularities emerge. One that has received attention previously (e.g., Eaton, Kortum, and Kramarz (2011), Eaton, Kortum, and Sotelo (2013)) is the tight connection between market size, market share, and the number of individual exporters. Figures 1 illustrates this relationship for French manufacturing exports to other members of the European Union (EU) in 2005.[1] Figure 1 reports a destination's market size, as measured by its manufacturing absorption, on the x-axis, and the number of French manufacturing firms selling there, on the y-axis. On a log-log scale, the slope is 0.52 (standard error 0.06).[2]

Combining data on French exporters and their buyers reveals new facts. Figure 2 plots the average number of buyers per French exporter across the other EU members, again with market size on the x-axis. The relationship is positive, with a slope of 0.34 (0.04), indicating another important margin of export expansion with market size. Nonetheless, as the y-axis in Figure 2 reveals, the average exporter has few buyers, motivating the granular theory of import demand we pursue in this paper.

These data also allow us to count the number of *relationships* between French exporters and their buyers in any destination. Regressing this measure on both market size and French

[1] Data sources are described in Appendix A.

[2] Figures 1-3 are all plotted on a log-log scale, and the corresponding regressions are run in logs (with standard errors of the coefficient estimates in parentheses).

market share (the share of a destination's absorption spent on manufacturing imports from France) we obtain a slope of 0.83 (0.06) on market size and 1.04 (0.16) on market share. Most striking, this last coefficient is indistinguishable from 1, inviting us to normalize the number of relationships by market share before plotting against market size. The result, in Figure 3, illustrates the very tight connection between relationships and the size of the destination market.

The small number of buyers in each destination shown in Figure 2 hides the extreme heterogeneity in the number reached by any given exporter. Table 1 reveals this underlying variation across French exporters to four EU destinations of diverse size. Note that the median number of buyers never exceeds 2 even in Germany, the largest EU market. But, numbers at the top end exceed one hundred.[3] Our model must account for this highly skewed distribution of buyers per firm.

Most existing theory has taken a monolithic approach to modeling technology, with all firms in a sector employing factors and intermediate inputs in the same way. But the data reveal substantial heterogeneity with respect to inputs as well. Figure 4 portrays the distribution of the share of production labor in total costs (including the cost of intermediates) across French manufacturing firms (the y-axis show the percent of firms with a share of production labor less than or equal to the value shown on the x-axis).

We seek to capture both the heterogeneity and the granularity in individual buyer-seller relationships in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who

[3] Our findings in Table 1 and Figure 2 on buyers per firm match evidence from Norwegian exporters reported in Bernard, Moxnes, and Ultveit-Moe (2016), their Figures 1 and 2 in particular.

may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

In contrast to standard production theory, we model a firm's technology as combining a set of tasks. Each task can be performed by labor, which can be of different types appropriate for different tasks. But labor competes with intermediate goods produced by other firms which can also perform these tasks. Firms may thus look very different from one another in terms of their production structure, depending on the sellers of intermediate goods that they happen to encounter. A firm's cost in a market thus depends not only on its underlying efficiency, but also on the costs of its suppliers. An implication is that an aggregate change, such as a reduction in trade barriers, can reduce the share of labor in production by exposing producers to more and cheaper sources of supply.

Our model is complementary to recent work of Oberfield (2013) in which a producer's cost depends not only on its own efficiency but the efficiencies of its upstream suppliers. It is also complementary to recent work of Chaney (2014) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), with trade the consequence of individual links formed between buyers and sellers over time. In order to embed the framework into general equilibrium, however, our analysis here remains static, more in line with the two-stage model of production in Bernard, Moxnes, and Ultveit-Moe (2016).[4] Our model also relates to Garetto (2013), in that firms

[4] Bernard, Moxnes, and Saito (2015) apply this model to micro data from Japan to evaluate the effects of a new high-speed train line on firms' supplier networks.

and workers compete directly to provide inputs for firms.[5]

We proceed as follows. Section 2 develops our model. Section 3 analyzes its implications for aggregate outcomes such as the distribution of wages. Section 4 turnss to firm-level outcomes such as the number of relationships between sellers in one country and buyers in another. Section 5 is an initial peak at the model's quantitative implications. Section 6 concludes.

2 A Model of Production through Random Encounters

Consider a world with a set of $i = 1, 2, ..., \mathcal{N}$ countries. Each country has an endowment of L_i^l workers of type $l = 1, 2, ..., L$.

2.1 Technology

A producer j in country i can make a quantity of output $Q_i(j)$ by combining a discrete set of tasks, $k = 1, ..., K$. Any task k consists of n_k subtasks, indexed by ω (which we refer to as a k-subtask). The production function, for a firm with numbers of k-subtasks given by $\{n_k\}_{k=1}^K$,

[5] In addition to the work already cited, our paper relates closely to a number of active areas. One is recent work on exports and the labor market, including Caliendo and Rossi-Hansberg (2012), Egger and Kreickemeier (2009), Felbermayr, Prat, and Schmerer (2008), Helpman, Itskhoki, and Redding (2010), and Hummels, Jørgenson, Munch, and Xiang (2014). Another is quantitative work focussing on firm-level imports, including Biscourp and Kramarz (2007), Blaum, Lelarge, and Peters (2014), Bricongne, Lionel, Gaulier, Taglioni, and Vicard (2012), Caliendo, Monte, and Rossi-Hansberg (2015), Frías, Kaplan, and Verhoogen (2009), Helpman, Itskhoki, Muendler, and Redding (2012), Irarrazabal, Moxnes, and Ulltveit-Moe (2013), Klein, Moser, and Urban (2010), Kramarz (2009), and Kramarz, Martin, and Mejean (2015). A third is other theories of networks or input-output interactions, including Acemoglu and Autor (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Lucas (2009), and Luttmer (2015).

is:

$$Q_i(j;\{n_k\}) = z_i(j) \prod_{k=1}^{K} b_k \left(\frac{1}{\beta_k} \left(\sum_{\omega=1}^{n_k} x_{k,i}(j,\omega)^{(\sigma_k-1)/\sigma_k} \right)^{\sigma_k/(\sigma_k-1)} \right)^{\beta_k}, \tag{1}$$

where $z_i(j)$ is the overall efficiency of producer j, $x_{k,i}(j,\omega)$ is the input of k-subtask ω, σ_k is the elasticity of substitution between k-subtasks, b_k is a constant (which we use later to neutralize the effect of n_k on task k productivity), and β_k is the Cobb-Douglas share of task k, satisfying $\beta_k > 0$ and

$$\sum_{k=1}^{K} \beta_k = 1.$$

An important special case of our framework assumes that $n_k = 1$ for all tasks k and for all producers j. In that case (1) reduces to a Cobb-Douglas production function. The point of introducing more than a single subtask ω is to more flexibly match our data on firm-to-firm trade. Allowing heterogeneity across firms along this dimension captures the observation that some firms have a very large number of suppliers. An elasticity of substitution greater than one among subtasks allows for the possibility that buyers purchase more when transacting with more efficient sellers.

Consider k-subtask ω. It can be performed either by the unique type of labor appropriate for that task, denoted $l(k)$, or with an input produced by a firm. We allow $K \geq L$, so that one type of labor might be able to perform tasks of several different types. We denote the set of tasks that labor of type l can perform as Ω_l. Worker productivity performing k-subtask ω for a given firm is $q_{k,i}(j,\omega)$. If the firm hires labor it pays the wage for workers of type $l(k)$. The producer also is in contact with a set of suppliers of an intermediate good that can also perform the subtask. From producer j's perspective, labor and the available inputs are perfect substitutes for performing any subtask. Hence it chooses whatever performs the subtask at lowest cost.

We assume that producers can hire labor in a standard Walrasian labor market at the market wage $w_{k,i} = w_i^{l(k)}$. In finding intermediates, however, buyers match with only an integer number of potential suppliers, either because of search frictions or because only a handful of producers make an input appropriate for this particular firm. We could make various assumptions about the price at which the intermediate is available. Because it yields the simplest set of results, we assume Nash bargaining in which the buyer has all the bargaining power, so that the price is pushed down to unit cost.[6]

Let $c_{k,i}^{\min}(j,\omega)$ denote the lowest price available to firm j for an intermediate to perform k-subtask ω. The price it pays to perform this subtask is thus:

$$c_{k,i}(j,\omega) = \min\left\{\frac{w_{k,i}}{q_{k,i}(j,\omega)}, c_{k,i}^{\min}(j,\omega)\right\}$$

and the firm's unit cost of delivering a unit of its output to destination n is:

$$c_{ni}(j;\{n_k\}) = \frac{d_{ni}}{z_i(j)} \prod_{k=1}^{K} \frac{1}{b_k} \left[\left(\sum_{\omega=1}^{n_k} c_{k,i}(j,\omega)^{-(\sigma_k-1)}\right)^{-(\sigma_k-1)}\right]^{\beta_k}, \tag{2}$$

where $d_{ni} \geq 1$ is the iceberg transport cost of delivering a unit of output from source i to destination n, with $d_{ii} = 1$ for all i.

In order to derive a closed form solution we impose specific distributions for producer efficiency, the efficiency of labor in performing a task, and the distribution of the prices of intermediate inputs.

[6] An implication is that there are no variable profits. Our model thus cannot accommodate fixed costs, either of market entry as in Melitz (2003) or in accessing markets for inputs, as in Antras et al. (2014). An alternative, which would allow for variable profits and hence fixed costs, is Bertrand pricing. While we found this alternative analytically tractable, we deemed the added complexity not worth the benefit.

First, following Melitz (2003) and Chaney (2008), each country has a measure of potential producers. The measure of potential producers in country i with efficiency $z_i(j) \geq z$ is:

$$\mu_i^Z(z) = T_i z^{-\theta}, \tag{3}$$

where $T_i \geq 0$ is a parameter reflecting the magnitude of country i's endowments of technology and $\theta \geq 0$ their similarities.

Second, worker productivity performing a task for a given producer $q_{\omega,i}(j)$ is a random variable Q drawn from the distribution:

$$F(q) = \Pr[Q \leq q] = e^{-q^{-\phi}}, \tag{4}$$

where $\phi \geq 0$ reflects the similarity of labor productivities across tasks and firms. For purposes that will become apparent below we restrict $\phi \leq \theta$.

Third, the measure of potential producers from i who can produce a good at a unit cost below c is given by:

$$\mu_{ii}(c) = T_i \Xi_i c^{\theta}, \tag{5}$$

where $\Xi_i \geq 0$. It follows that the measure of potential producers from i who can deliver to n at a cost below c is:

$$\mu_{ni}(c) = \mu_{ii}(c/d_{ni}) = d_{ni}^{-\theta}\mu_{ii}(c) = T_i \Xi_i d_{ni}^{-\theta} c^{\theta}.$$

Our specifications of the heterogeneity in producer efficiency given in (3) and the distribution of labor productivity given in (4) are primitives of the model, with T_i, θ, and ϕ exogenous parameters. We show below, however, that the resulting heterogeneity in unit costs c given by (5) arises endogenously from our other assumptions, with Ξ_i determined by underlying technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

A potential producer only survives if it meets a customer who buys from it. A customer could be a final consumer (a household) or a firm which uses the producer's output as an input. The measure of final consumers in market n is the exogenous measure of households:

$$L_n = \sum_l L_n^l.$$

The measure of *active* producers in market n is determined endogenously by the *potential* producers there that are able to make a sale (either in market n or in some other destination).

2.2 Preferences

Final demand is by households supported by workers of different skill levels spending their wage income (since there are no profits in our model). Analogous with the tasks that firms need to perform, households have an integer number of needs indexed by k, that are combined with a Cobb-Douglas share α_k. We thus treat consumers in parallel to firms as buyers. In particular, like a producer's tasks, a household's needs can be fulfilled either by goods purchased from a producer or directly by labor.

2.3 Matching Buyers and Sellers

In contrast with standard Walrasian models, we assume that matching between buyers and sellers is random. Even though there are a continuum of possible sellers and buyers, an individual seller matches with only an integer number of potential buyers and an individual buyer matches with only an integer number of potential sellers. The matching literature (e.g., Mortensen and Pissarides, 1994) typically posits that in a market with more potential buyers and sellers, the likelihood of a match between any given potential buyer and potential seller is smaller.[7]

In our case, however, the measure of potential sellers implied by (5) is unbounded. But for a seller with unit cost c, the measure of sellers with unit cost below c is always bounded. So instead we treat the likelihood of a match involving a seller with unit cost c as limited by the measure of sellers with unit cost below c.

Matching frictions in the model are captured by the parameters $\lambda_{k,ni}$, which governs the rate of contact between sellers in i and buyers in n seeking to perform a k-subtask. For tractability we impose the following separability assumption:

$$\lambda_{k,ni} = \lambda_k \lambda_{ni}.$$

We thus posit that the intensity with which a seller in i with unit cost c in delivering to country n encounters a buyer seeking to fulfill a k-subtask is:

$$e_{k,ni}(c) = \lambda_k \lambda_{ni} L_n^{-\varphi} \mu_n(c)^{-\gamma}, \tag{6}$$

where:

$$\mu_n(c) = \sum_i \lambda_{ni} \mu_{ni}(c).$$

The key new parameters are (i) $\varphi \geq 0$, which governs how other buyers impede the ability of a given buyer to match with a seller and (ii) $0 \leq \gamma < 1$, which governs how lower cost sellers impede the ability of a seller to match with a buyer.

[7] Matching in our framework can be interpreted literally as coming into contact with each other, but it also could relate to the appropriateness of a seller's product for the buyer's purpose. In this sense we can think of products as differentiated not only by seller, but by buyer as well.

Aggregating across potential suppliers from each source i, with different costs of delivering to n, the number of "quotes" that a buyer in n receives for a k-subtask with a price below c is distributed Poisson with parameter:

$$
\begin{aligned}
\rho_{k,n}(c) &= \sum_i \int_0^c e_{k,ni}(x) d\mu_{ni}(x) \\
&= \lambda_k L_n^{-\varphi} \int_0^c \mu_n(x)^{-\gamma} \sum_i \lambda_{ni} d\mu_{ni}(x) \\
&= \lambda_k L_n^{-\varphi} \int_0^c \mu_n(x)^{-\gamma} d\mu_n(x) \\
&= \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \mu_n(c)^{1-\gamma}.
\end{aligned}
$$

This Poisson parameter grows arbitrarily large with c, so that many potential suppliers are available to serve any given buyer.

We now consider the measure of matches between customers in country n seeking to fulfill a k-subtask and sellers with unit cost below c when selling in n. This measure is simply the product of the measure of firms buying in country n and the expected number of suppliers per buyer for k-subtasks:

$$
m_{k,n}(c) = (L_n + M_n)\, \rho_{k,n}(c) = \frac{\lambda_k E_k}{1-\gamma} (L_n + M_n)\, L_n^{-\varphi} \mu_n(c)^{1-\gamma},
$$

where

$$
E_k = \sum_{n_k=1}^{\infty} P(n_k) n_k
$$

is the expected number of k-subtasks per buyer ($P(n_k)$ is the probability of a buyer having exactly n_k subtasks of task k).

The firm can perform a k-subtask at a cost below c_k unless the cost of hiring workers directly and the lowest quote both exceed c_k. From the Poisson density, we know that with probability $\exp\left[-\rho_{k,n}(c_k)\right]$ the buyer will encounter no quotes below c_k. It will cost more than c_k to hire workers to perform the task if $w_{k,n}/Q > c_k$, which occurs with probability $F(w_{k,n}/c_k)$. Since the two events are independent the distribution of the lowest cost to fulfill a k-subtask is:

$$G_{k,n}(c_k) = 1 - F(w_{k,n}/c_k)e^{-\rho_{k,n}(c_k)}.$$

To work out the implications of this distribution for the resulting distribution of production costs, we restrict:

$$\gamma = \frac{\theta - \phi}{\theta}.$$

With this restriction, the parameter governing heterogeneity in the distribution of costs of intermediates is the same as the parameter governing heterogeneity in the distribution of worker efficiency (4) at a given k-subtask for a given buyer. In particular, the distribution of the cost to the buyer of fulfilling a k-subtask becomes:

$$G_{k,n}(c_k) = 1 - e^{-\Xi_{k,n} c_k^{\phi}}, \tag{7}$$

where

$$\Xi_{k,n} = \nu_{k,n} + w_{k,n}^{-\phi} \tag{8}$$

and

$$\nu_{k,n} = \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{1-\gamma}, \tag{9}$$

where

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i.$$

Hence

$$\mu_n(c) = \sum_i \lambda_{ni}\mu_{ni}(c) = \sum_i \lambda_{ni} T_i \Xi_i d_{ni}^{-\theta} c^{\theta} = \Upsilon_n c^{\theta}.$$

2.4 Preliminary Results

At this point we can already derive results about labor shares and trade shares.

With probability $w_{k,n}^{-\phi}/\Xi_{k,n}$ the buyer hires workers to perform a k-subtask while with probability $\nu_{k,n}/\Xi_{k,n}$ it purchases an intermediate from the lowest-cost supplier. Notice that these probabilities are independent of the unit cost c.

While $w_{k,n}^{-\phi}/\Xi_{k,n}$ is the probability that a k-subtask is performed by labor in country n, since there are a continuum of producers, it is also the aggregate share of labor in performing task k in country n.[8] The aggregate share of labor of type l in total production costs is consequently:

$$\beta_n^l = \sum_{k \in \Omega_l} \beta_k w_{k,n}^{-\phi}/\Xi_{k,n}$$

and the overall labor share in production costs is:

$$\beta_n^L = \sum_l \beta_n^l.$$

Note that, even though our basic technology is Cobb-Douglas across tasks k, the labor share depends on wages and other factors.

To derive bilateral trade shares, given factor prices, we need to solve for $\rho_{k,ni}(c)$, the Poisson parameter for the number of "quotes" from a potential supplier in i that a buyer in n receives for a k-subtask with a price below c. This parameter is:

$$\rho_{k,ni}(c) = \int_0^c e_{k,ni}(x) d\mu_{ni}(x) = \frac{\lambda_k}{1-\gamma} L_n^{-\varphi} \Upsilon_n^{-\gamma} \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i c^{\phi}.$$

[8] Similarly, in Eaton and Kortum (2002) the probability π_{ni} that destination n buys a good from a source i is also source i's share in destination n's spending.

Thus, the bilateral trade share is:

$$\pi_{ni} = \frac{\rho_{k,ni}(c)}{\rho_{k,n}(c)} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\Upsilon_n} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\sum_{i'} \lambda_{ni'} d_{ni'}^{-\theta} T_{i'} \Xi_{i'}}.$$

Just as in Eaton and Kortum (2002), with our continuum of producers, in the aggregate π_{ni} is the share of source i in the purchases of destination n. At this point we can't fully interpret this equation, as we have not yet derived Ξ_i.

We proceed by showing first how the cost measure (5) arises from our model of firm-to-firm trade. We then turn to consumer demand and then to intermediate demand before closing the model in general equilibrium.

2.5 Deriving the Cost Distribution

Each c_k is distributed independently according to (7). From (3) and (2), the measure of potential producers from source i that can produce at a unit cost below c is:

$$\begin{aligned} \mu_{ii}(c) &= T_i c^{\theta} \prod_k b_k^{\theta} \left[\int_0^{\infty} \cdots \int_0^{\infty} \left(\sum_{\omega=1}^{n_k} c_{\omega}^{-(\sigma_k-1)} \right)^{\theta\beta_k/(\sigma_k-1)} dG_{k,i}(c_1) \ldots dG_{k,i}(c_{n_k}) \right] \\ &= T_i \Xi_i c^{\theta} \end{aligned} \tag{10}$$

where:

$$\Xi_i = \prod_k b_k^{\theta} \Psi_{k,i}$$

and

$$\begin{aligned} \Psi_{k,i} &= \int_0^{\infty} \cdots \int_0^{\infty} \left(\sum_{\omega=1}^{n_k} c_{\omega}^{-(\sigma_k-1)} \right)^{\theta\beta_k/(\sigma_k-1)} \phi \prod_{\omega=1}^{n_k} \Xi_{k,i} c_{\omega}^{\phi-1} e^{-\Xi_{k,i} c_{\omega}^{\phi}} dc_{\omega} \\ &= \int_0^{\infty} \cdots \int_0^{\infty} \left(\sum_{\omega=1}^{n_k} \left(\frac{x_{\omega}}{\Xi_{k,i}} \right)^{-(\sigma_k-1)/\phi} \right)^{\theta\beta_k/(\sigma_k-1)} \prod_{\omega=1}^{n_k} e^{-x_{\omega}} dx_{\omega} \\ &= \int_0^{\infty} e^{-x_1} \cdots \int_0^{\infty} e^{-x_{n_k-1}} \left[\int_0^{\infty} e^{-x_{n_k}} \left(\sum_{\omega=1}^{n_k} \left(\frac{x_{\omega}}{\Xi_{k,i}} \right)^{-(\sigma_k-1)/\phi} \right)^{\theta\beta_k/(\sigma_k-1)} dx_{n_k} \right] dx_{n_k-1} \ldots dx_1, \end{aligned}$$

where in the second and third line we have changed the variables of integration to $x_\omega = \Xi_{k,i} c_\omega^\phi$.

To simplify further, we impose the restriction that $\theta\beta_k = \sigma_k - 1$, giving us:

$$\begin{aligned}
\Psi_{k,i} &= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k - 1}} \left[\int_0^\infty e^{-x_{n_k}} \left(\sum_{\omega=1}^{n_k} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} \right) dx_{n_k} \right] dx_{n_k - 1} ... dx_1 \\
&= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k - 1}} \left[\sum_{\omega=1}^{n_k - 1} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} + \int_0^\infty e^{-x_{n_k}} \left(\frac{x_{n_k}}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} dx_{n_k} \right] dx_{n_k - 1} ... dx_1 \\
&= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{n_k - 2}} \left[\int_0^\infty e^{-x_{n_k - 1}} \left(\sum_{\omega=1}^{n_k - 1} \left(\frac{x_\omega}{\Xi_{k,i}} \right)^{-\theta\beta_k/\phi} \right) dx_{n_k - 1} \right] dx_{n_k - 2} ... dx_1 \\
&\quad + \Gamma\left(1 - \theta\beta_k/\phi\right) \Xi_{k,i}^{\theta\beta_k/\phi}, \qquad (11)
\end{aligned}$$

where we require that parameter values satisfy $\theta\beta_k/\phi < 1$.

Since the remaining integral has the same form as what we started with, we can repeat the steps above again and again to obtain:

$$\Psi_{k,i} = n_k \Gamma\left(1 - \theta\beta_k/\phi\right) \Xi_{k,i}^{\theta\beta_k/\phi}.$$

Setting

$$b_k = \left[n_k \Gamma\left(1 - \theta\beta_k/\phi\right) \right]^{-1/\theta},$$

we have:

$$\Xi_i = \prod_k b_k^\theta \Psi_{k,i} = \prod_k \Xi_{k,i}^{\theta\beta_k/\phi}.$$

We can either solve for the vector of Ξ_i from the system of equations:

$$\Xi_i = \prod_{k=1}^K \left(\frac{\lambda_k}{1-\gamma} L_i^{-\varphi} \left(\sum_{i'} \lambda_{ii'} d_{ii'}^{-\theta} T_{i'} \Xi_{i'} \right)^{1-\gamma} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}$$

or, substituting in (9), we can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \prod_k \left(\frac{\lambda_k}{1-\gamma} L_i^{-\varphi} \Upsilon_i^{1-\gamma} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k} \qquad (12)$$

for $n = 1, 2, ..., \mathcal{N}$. Given wages and exogenous parameters of the model, the Υ_n are thus the solution to the set of equations (12). Appendix B provides sufficient conditions for a unique solution to the Υ_n's and an iterative procedure to compute them.

2.6 Households

Final demand is by different types of workers spending their wage income (since there are no profits in our model). As mentioned above, we model their preferences in parallel to our assumptions about production. Consumers have an integer number K of needs, with each need having a Cobb-Douglas share α_k in preferences, with $\alpha_k > 0$ and

$$\sum_{k=1}^{K} \alpha_k = 1.$$

Proceeding as above, a consumer faces a distribution of costs for fulfilling each subtask of need k given by (7). The probability that a k-subtask is fulfilled by labor is again $\upsilon_{k,i}$, which, with our continuum of consumers, is the share of labor in fulfilling need k. The share of labor of type l used by consumers in their total spending is thus:

$$\alpha_i^l = \sum_{k \in \Omega_l} \alpha_k \upsilon_{k,i}$$

and the share of labor in consumer spending in country i is:

$$\alpha_i^L = \sum_l \alpha_i^l.$$

As with the share of labor in production costs, the share of labor in final spending depends on wages and other factors.

When a consumer in country n fulfills a k-subtask by purchasing a good, the probability that the good come from country i is given by π_{ni} in expression (**??**). With our continuum of consumers π_{ni} thus represents the share of country i in country n's final spending.

2.7 Consumer Welfare

Two workers with the same income won't typically have the same level of utility, as they encounter different goods and worker productivities in satisfying their needs. The income (hence expenditure) $Y(V)$ needed to obtain expected utility V in market n, for an individual with needs fulfilled by numbers of k-subtasks given by $\{n_k\}_{k=1}^{K}$, is thus:

$$Y(V;\{n_k\}) = V\prod_k \frac{1}{a_k}\left[\int_0^\infty ... \int_0^\infty \left(\sum_{\omega=1}^{n_k} c_\omega^{-(\sigma_k-1)}\right)^{-\alpha_k/(\sigma_k-1)} dG_{k,n}(c_1)...dG_{k,n}(c_{n_k})\right].$$

In parallel to the derivation of the cost distribution (11), we can simplify this expression to obtain:

$$Y(V;\{n_k\}) = V\prod_k \frac{1}{a_k} n_k \Gamma\left(1+\alpha_k/\phi\right)\Xi_{k,n}^{-\alpha_k/\phi}.$$

To neutralize the effect of n_k, we set $a_k = n_k\Gamma\left(1+\frac{1}{\phi}\alpha_k\right)$ so that the expected expenditure function becomes:

$$Y(V) = V\prod_{k=1}^{K}\left(\Xi_{k,n}\right)^{-\frac{1}{\phi}\alpha_k}.$$

We can write the result more compactly as:

$$Y(V) = P_n^C V,$$

where

$$P_n^C = \prod_{k=1}^{K}\left(\Xi_{k,n}\right)^{-\frac{1}{\phi}\alpha_k}$$

is the consumer price index.

3 Aggregate Equilibrium

We now have in place the assumptions we need to solve for the aggregate equilibrium. We first solve for equilibrium in the production of intermediates, given wages, and then turn to labor-market equilibrium, which determines those wages.

3.1 Production Equilibrium

With balanced trade, total final spending X_n^C is labor income:

$$X_n^C = \sum_{l=1}^{L} w_n^l L_n^l = \sum_{k=1}^{K} w_{k,n} L_{k,n}. \tag{13}$$

Total production in country i equals total revenue in supplying consumption goods and intermediates around the world:

$$Y_i = \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[\Phi_n^C X_n^C + \Phi_n^I Y_n \right]$$

where $\Phi_n^C = 1 - \alpha_n^L$ and $\Phi_n^I = 1 - \beta_n^L$, the shares of goods in final spending and in production spending, respectively.

We can write this result in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{\Pi} \left(\boldsymbol{\Phi}^C \mathbf{X}^C + \boldsymbol{\Phi}^I \boldsymbol{Y} \right)$$

where:

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_{\mathcal{N}} \end{bmatrix}, \quad \mathbf{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ \cdot \\ \cdot \\ \cdot \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$\mathbf{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & \dots & 0 & 0 \\ 0 & \Phi_2^j & \dots & 0 & 0 \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ 0 & 0 & \dots & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & \dots & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I$$

and:

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & \dots & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & \dots & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ . & . & \cdot\ \cdot\ \cdot & . & . \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}$$

We can then solve for $\boldsymbol{Y}$:

$$\boldsymbol{Y} = (\boldsymbol{I}_{\mathcal{N}} - \mathbf{\Pi}\mathbf{\Phi}^I)^{-1}\mathbf{\Pi}\mathbf{\Phi}^C\mathbf{X}^C$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

3.2 Labor-Market Equilibrium

With balanced trade, final spending in country i, X_i^C is given by (13). Equilibrium in the market for labor of type l in country i solves the expression:

$$w_i^l L_i^l = \alpha_i^l X_i^C + \beta_i^l Y_i.$$

where the first term on the right-hand side corresponds to labor demanded directly by households and the second term to labor demanded by firms. These sets of equations, for each type of labor l in each country i, determine the wage w_i^l.

4 Implications for Individual Producers

While our analysis so far has allowed us to investigate the implications of various changes in exogenous variables on equilibrium aggregate outcomes, we have more work to do to find out what happens to individual producers. We start by calculating the measure of producers active in each market. We then turn to the number of firms selling in each market and the number of relationships, as discussed in the introduction.

4.1 Active Producers

A producer is active as long as it has a customer that buys from it. How many buyers a firm has depends not only on its efficiency z, but on its luck in finding low-cost suppliers and its luck in running into buyers who don't have better alternatives.

Consider a supplier from i with unit cost c in market n. The number of buyers, households or other firms, that it connects with, seeking to carry out a k-subtask, is distributed Poisson with parameter:

$$\begin{aligned} E_k\left(L_n+M_n\right)e_{k,ni}(c) &= E_k\left(L_n+M_n\right)\lambda_k\lambda_{ni}L_n^{-\varphi}\mu_n(c)^{-\gamma} \\ &= \left(L_n+M_n\right)E_k\lambda_k\lambda_{ni}L_n^{-\varphi}\left(\Upsilon_n c^{\theta}\right)^{-\gamma}, \end{aligned}$$

where recall that E_k is the expected number of tasks of type k across final and intermediate buyers.

Having met a buyer, this supplier will make the sale with probability $e^{-\Xi_{k,n}c^{\phi}}$, the probability that that there is no lower quote. Combining these two results the number of customers in n buying from a supplier from i with unit cost c for a k-subtask is distributed Poisson with parameter $\eta_{k,ni}(c)$, given by:

$$\eta_{k,ni}(c) = (L_n + M_n)\, E_k \lambda_k \lambda_{ni} L_n^{-\varphi} \left(\Upsilon_n c^{\theta}\right)^{-\gamma} e^{-\Xi_{k,n} c^{\phi}},$$

where, recall,

$$\Xi_{k,n} = \nu_{k,n} + w_{k,n}^{-\phi}.$$

Note that $\eta_{k,ni}(c)$ is decreasing in the producer's unit cost c for two reasons. First, as long as $\gamma > 0$, a low-cost producer typically finds more potential customers. Second, each potential customer is more likely to have no better option. Note also that, given Υ_n and $w_{k,n}$, the Poisson parameter is at first increasing and then decreasing in λ_k. If it's impossible to meet customers ($\lambda_k = 0$) then it's impossible to make a sale. Thus, starting from 0, an increase in λ_k increases the likelihood of a sale. But an increase in λ_k also means that a potential buyer is more likely to have found another seller with a lower cost. At some point (which is earlier for a firm with a high c), as λ_k rises, this second effect dominates, so that further increases reduce expected sales.

Since purchases are independent across k, the total number of customers in n for a producer in i with unit cost c is distributed Poisson with parameter:

$$\eta_{ni}(c) = \sum_{k=1}^{K} \eta_{k,ni}(c) = \lambda_{ni} (L_n + M_n)\, L_n^{-\varphi} \left(\Upsilon_n c^{\theta}\right)^{-\gamma} \sum_{k=1}^{K} E_k \lambda_k e^{-\Xi_{k,n} c^{\phi}}. \tag{14}$$

By the properties of the Poisson distribution, $\eta_{ni}(c)$ is also the expected number of customers for a potential producer from i selling a product at unit cost c in market n.

Consider customers worldwide for a producer in country i with local cost c. Its unit cost in country n is cd_{ni}. The total number of customers around the world for this producer is distributed Poisson with parameter:

$$\begin{aligned}\eta_i^W(c) &= \sum_{n=1}^{\mathcal{N}} \eta_{ni}(cd_{ni}) \\ &= \sum_{n=1}^{\mathcal{N}} \lambda_{ni} d_{ni}^{-\gamma\theta} \left(L_n + M_n\right) L_n^{-\varphi} \left(\Upsilon_n c^\theta\right)^{-\gamma} \sum_{k=1}^{K} E_k \lambda_k e^{-\Xi_{k,n} d_{ni}^{-\phi} c^\phi}.\end{aligned}$$

Thus, the probability that a potential producer from source i with unit cost c fails to make a sale anywhere is $\exp(-\eta_i^W(c))$.

Integrating over the cost distribution of potential producers in source i (those from i that can deliver to i at cost c):

$$\begin{aligned}M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\ &= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc. \end{aligned} \tag{15}$$

Since $\eta_i^W(c)$ itself depends on the measure of customers for intermediates M_n in each market n, we need to iterate to find a solution for all the M_i's.

4.2 The Measure of Sellers

Having solved for the measure of active producers in each location, we can now calculate the theoretical analogs of the firm-level statistics that we can observe in our data. Since the data record customers as firms, we ignore households (final consumers) in these calculations. Thus, we work with a slight modification of equation (14) that drops L_n from the measure of buyers:

$$\eta_{ni}^M(c) = \lambda_{ni} M_n L_n^{-\varphi} \left(\Upsilon_n c^\theta\right)^{-\gamma} \sum_{k=1}^{K} E_k \lambda_k e^{-\Xi_{k,n} c^\phi},$$

where M_n is given by (15).The probability that a producer in country i will have at least one firm customer in n, if it can sell there at cost c, is simply $1 - \exp(-\eta_{ni}^M(c))$. We can calculate

N_{ni}, the measure of firms from i selling in n, by integrating over the cost distribution:

$$\begin{aligned} N_{ni} &= \int_0^\infty (1 - e^{-\eta_{ni}^M(c)}) d\mu_{ni}(c) \\ &= T_i \Xi_i d_{ni}^{-\theta} \int_0^\infty (1 - e^{-\eta_{ni}^M(c)}) \theta c^{\theta-1} dc. \end{aligned} \tag{16}$$

We need to evaluate this integral numerically. The total measure of producers selling to firms in n is:

$$N_n = \sum_{i=1}^{\mathcal{N}} N_{ni}.$$

4.3 The Measure of Relationships

So far we've considered whether or not a firm has any customers in a market, but not how many. We refer to the total number of firm seller-buyer transactions as the number of relationships. The measure of relationships between sellers in i and firm buyers in n is:

$$\begin{aligned} R_{ni} &= \int_0^\infty \eta_{ni}^M(c) d\mu_{ni}(c) = T_i \Xi_i d_{ni}^{-\theta} \int_0^\infty \eta_{ni}^M(c) \theta c^{\theta-1} dc \\ &= \pi_{ni} \Upsilon_n M_n L_n^{-\varphi} \Upsilon_n^{-\gamma} \int_0^\infty \sum_{k=1}^K E_k \lambda_k c^{-\gamma\theta} e^{-\Xi_{k,n} c^\phi} \theta c^{\theta-1} dc \\ &= \pi_{ni} M_n \sum_{k=1}^K \nu_{k,n} E_k \left(1 - \gamma\right) \int_0^\infty e^{-\Xi_{k,n} c^\phi} \theta c^{\phi-1} dc \\ &= \pi_{ni} M_n \sum_{k=1}^K \nu_{k,n} E_k \left. \frac{-1}{\Xi_{k,n}} e^{-\Xi_{k,n} c^\phi} \right|_0^\infty \\ &= \pi_{ni} M_n \sum_{k=1}^K E_k \frac{\nu_{k,n}}{\Xi_{k,n}}. \end{aligned}$$

Thus, the measure of relationships is simply the product of the number of active producers in n, the sum across tasks of the expected number of outsourced k-subtasks ($E_k \nu_{k,n} / \Xi_{k,n}$), and the share of spending in n going to sellers from i. The total measure of relationships involving buyers from destination n is:

$$R_n = \sum_i R_{ni} = M_n \sum_{k=1}^{K} E_k \frac{\nu_{k,n}}{\Xi_{k,n}}.$$

4.4 The Distribution of Buyers

We now turn to the distribution of the number of relationships. Let S_{ni} be the integer-valued random variable for the number of firm buyers in n for a firm from i selling in n at cost c. From the Poisson distribution:

$$\Pr[S_{ni} = s|c] = \frac{e^{-\eta_{ni}^M(c)} \left[\eta_{ni}^M(c)\right]^s}{s!},$$

for $s = 0, 1, ...$ For any $s \geq 1$, we can integrate over the cost measure to obtain the measure of firms from i with s firm buyers in n:

$$\begin{aligned} N_{ni}(s) &= \int_0^\infty \frac{e^{-\eta_{ni}^M(c)} \left[\eta_{ni}^M(c)\right]^s}{s!} d\mu_{ni}(c) \\ &= \frac{T_i \Xi_i d_{ni}^{-\theta}}{s!} \int_0^\infty e^{-\eta_{ni}^M(c)} \left[\eta_{ni}^M(c)\right]^s \theta c^{\theta-1} dc. \end{aligned}$$

Note that the measure of firms from i with at least one firm buyer in n, given by (16), can also be expressed as:

$$N_{ni} = \sum_{s=1}^{\infty} N_{ni}(s).$$

Thus, the fraction of firms from i selling in n who have s firm buyers is $N_{ni}(s)/N_{ni}$, for $s = 1, 2, ...$ The number of relationships can now be expressed as:

$$R_{ni} = \sum_{s=1}^{\infty} s N_{ni}(s).$$

The expected number of customers per firm from i selling in n is:

$$E\left[S_{ni}|S_{ni} \geq 1\right] = \frac{R_{ni}}{N_{ni}} = \sum_{s=1}^{\infty} s \frac{N_{ni}(s)}{N_{ni}} = \frac{\pi_{ni}}{N_{ni}} M_n \sum_{k=1}^{K} E_k \frac{\nu_{k,n}}{\Xi_{k,n}}.$$

For any firm selling in n:

$$E\left[S_n | S_n \geq 1\right] = \frac{R_n}{N_n} = \frac{M_n}{N_n} \sum_{k=1}^{K} E_k \frac{\nu_{k,n}}{\Xi_{k,n}}.$$

4.5 Other Moments

In Eaton, Kortum, and Kramarz (2011) we focused on a number of conditional moments, such as the mean sales in France of French firms that export to market n. We now show how such moments can be computed from the model here.

The measure of relationships in i for firms from i that export to n is:

$$R_{ii|n} = \int_0^\infty \eta_{ii}^M(c) \left(1 - e^{-\eta_{ni}^M(cd_{ni})}\right) d\mu_{ii}(c).$$

Hence the expected number of buyers in i for a firm from i with at least one customer in n is:

$$E\left[S_{ii} | S_{ni} \geq 1\right] = \frac{R_{ii|n}}{N_{ni}}.$$

What we can in fact observe is the number of relationships in m for firms from i that also export to n:

$$R_{mi|n} = \int_0^\infty \eta_{mi}^M(cd_{mi}) \left(1 - e^{-\eta_{ni}^M(cd_{ni})}\right) d\mu_{ii}(c).$$

Hence the expected number of buyers in m for a firm from i with at least one customer in n is:

$$E\left[S_{mi} | S_{ni} \geq 1\right] = \frac{R_{mi|n}}{N_{ni}}.$$

We can look at this relationship across all n, for any given m (and i). The less common for firms in i to export to n, the stronger the selection (on low cost) for those that do. As a consequence, we expect such firms to find more buyers in m.

5 Some Quantitative Implications

We now investigate some quantitative implications of the model. Table 2 provides our parameterization. The parameter of the efficiency distribution is $\theta = 4$, which is standard in the literature. The parameter $\gamma = 0.5$ gives a substantial advantage to lower-cost firms in reaching buyers. By setting $E_k = 1$, firm-level production functions reduce to the Cobb-Douglas functional form.

The labor force in each country is divided into nonproduction workers (60 percent) and production workers (40 percent). Nonproduction workers can perform four tasks each with Cobb-Douglas shares $\beta^N = \alpha^N = 0.1$. Production workers can perform 24 tasks each with $\beta^P = \alpha^P = 0.025$. The key distinction between the nonproduction and production tasks is the extent to which they are subject to outsourcing. We set $\lambda^N = 0$ for each nonproduction task and $\lambda^P = 1.0$ for each production task.

The iceberg costs are relatively small, $d_{ni} = 1.2$ for all $i, n,\ i \neq n$ ($d_{ii} = 1$). But, bilateral contact rates of $\lambda_{ni} = 0.25$ for all $i, n,\ i \neq n$ ($\lambda_{ii} = 1$), also contribute to trade frictions. The world labor force, normalized at 1, is divided into six countries with the sizes given along the top of Table 3. The countries are identical to each other except for the sizes of their labor forces.

Note from Table 3 that the huge differences in the relative size of countries makes little difference for most outcomes. This result is a consequence of setting $\varphi = 0.2$ (if $\varphi = 0$ there would be strong scale effects due to the search technology). The one exception is the import share, which, as is typical in trade models, declines with country size. The iceberg costs together with bilateral contact rates lead to realistically low import shares in the larger countries.

The parameters were chosen to deliver a realistic share of production value added in GDP (about 13 percent). About 70 percent of tasks that could be outsourced are actually outsourced, so that production value added is only about 20 percent of gross production. Gross production itself rises roughly in proportion to a country's labor force.

We now turn to the firm-level implications in Table 4. The first two rows of the table show that the number of firms producing in a country and the number selling in a country rise with the size of the market. Relative to the labor force, however, the number of firms producing in a country differ little by country size. On the other hand, the number selling in a country, relative to the labor force there, fall dramatically with country size. This phenomenon is a reflection of the fact that the average exporter has more buyers in a larger market.

We can replicate the regression results reported in the introduction using our simulation results. To do so, we treat the second largest of the six countries as France. Regressing the number of French exporters on market size and French market share, we obtain a coefficient of 0.63 on market size and 0.61 on market share. These coefficients, both considerably below one, are not far from those obtained from the data. Similarly, regressing the number of French relationships on market size and French market share yields a coefficient of 0.94 on market size and 0.99 on market share. Note that in the model, as in the data, relationships move one-to-one with market share.

Returning to the last part of Table 4 shows the distribution of numbers of buyers across exporters. As in the data, the simulation displays a highly skew distribution. While the median number of buyers is one or two, the 99th percentile approaches 100. We ignore the second-to-last column, representing buyers in France, since we only observe buyers in foreign markets.

6 Conclusion

Taking into account the granularity of individual buyer-seller relationships expands the scope for firm heterogeneity in a number of dimensions. Aside from differences in raw efficiency, firms experience different luck in finding cheap inputs. These two sources of heterogeneity combine to create differences in the firm's cost to deliver to different markets around the world. But within each market firms have different degrees of luck in connecting with buyers. We can thus explain why a firm may happen to sell in a small, remote market while skipping over a large one close by. It also explains why one firm may appear very successful in one market and sell very little in another, while another firm does just the opposite.

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7 Appendix A: Data Source

The empirical analysis is conducted using detailed export data covering the universe of French exporting firms. The data have been provided by the French Customs, and have been used by Kramarz, Martin, and Mejean (2014). The full data set covers all export transactions that involve a French exporter and an importing firm located in the European Union. In this paper, we use only the data for the year 2005.

Many researchers before us have used individual trade data from the French Customs. Typically, the data used in such empirical analyses are annual measures disaggregated at the level of the exporting firm, as in Eaton, Kortum, and Kramarz (2011) among others. Some papers, such as Biscourp and Kramarz (2007) and Blaum, Lelarge, and Peters (2014), also use data at the level of the importer. An exception is Bricongne, Fontagné, Gaulier, Taglioni, and Vicard (2012) who use data that record, for each exporting firm, each transaction in each month, although not identifying the exact buyer. In this respect, the data we use are more precise since they not only record the transaction but also the exact identity of the buyer. For each transaction, the dataset gives us the identity of the exporting firm (its name and its SIREN identifier), the identification number of the importer (an anonymized version of its VAT number), the date of the transaction (month and year), the product category of the transaction (at the 8-digit level of the combined nomenclature), the value and the quantity of the shipment. For the analysis here, records will be aggregated across transactions within a year, for each exporter-importer-product triplet. Such measurement is possible because, whereas goods are perfectly free to move across countries within the European Union, firms selling goods outside France are still compelled to fill a Customs form. Such forms are used to repay VAT for transactions on intermediate consumptions. Hence, our data are exhaustive. However, small exporters are allowed to fill a "simplified" form that does not require the product category of exported goods. The "simplified" regime can be used by firms with total exports in the EU below 100,000 euros in 2005 (and 150,000 euros thereafter). In 2005, we have data for 46,928 French firms exporting 7,807 8-digit products to 571,149 buyers located in the EU. Total exports by these firms amounts to 207 billions of euros. Such exports account for 58 percent of French total exports. The total number of observations is 3,983,909.

8 Appendix B: Computing Υ

[***THIS APPENDIX IS NOT UPDATED***] We derive conditions under which there is a unique solution for Υ, given wages, that can be computed by simple iteration. To ensure a solution it helps to have a sufficient share of tasks in which outsourcing is not possible ($\lambda_k = 0$). Denote the set of such tasks as Ω^0 and its complement (among the set of all tasks $\{1, 2, ..., K\}$) as Ω^P. We require:

$$\beta^P = \sum_{k \in \Omega^P} \beta_k < 1.$$

As a warm-up exercise, we start with the case of a single country ($\mathcal{N} = 1$), so that Υ is a scalar. We then turn to the general case with multiple countries, in which Υ is an $\mathcal{N} \times 1$ vector.

8.1 The Case of a Single Country

With a single country, the solution for Υ is a fixed point

$$\Upsilon = f(\Upsilon)$$

of the function f defined as:

$$f(x) = T \prod_{k=1}^{K} \left(\frac{\theta}{\phi} \lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}.$$

Employing our assumption that $\lambda_k = 0$ for all tasks $k \in \Omega^0$, we can write:

$$f(x) = T \left(\prod_{k\in\Omega^0} (w_k)^{-\theta\beta_k} \right) \prod_{k\in\Omega^P} \left(\frac{\theta}{\phi} \lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}.$$

It is convenient to work in logs. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the function:

$$F(y) = A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right),$$

where

$$A = \ln T - \sum_{k\in\Omega^0} \theta \beta_k \ln w_k,$$

and

$$u_k = \frac{\theta}{\phi} \lambda_k$$

There exists a unique fixed point of F if it is a contraction. To show that it is, we can check Blackwell's sufficient conditions, monotonicity and discounting. For monotonicity, note that $x \leq y$ implies:

$$F(x) = A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} x} + w_k^{-\phi} \right) \leq A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) = F(y).$$

For discounting, $a > 0$ implies:

$$\begin{aligned} F(y+a) &= A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta}(y+a)} + w_k^{-\phi} \right) = A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(e^{\frac{\phi}{\theta} a} u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) \\ &= A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \left[\frac{\phi}{\theta} a + \ln \left(u_k e^{\frac{\phi}{\theta} y} + e^{-\frac{\phi}{\theta} a} w_k^{-\phi} \right) \right] = A + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + e^{-\frac{\phi}{\theta} a} w_k^{-\phi} \right) \\ &\leq A + \sum_{k\in\Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) + \beta^P a = F(y) + \beta^P a. \end{aligned}$$

We can thus compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with $y^{(0)} = 0$. This method is justified, since the contraction mapping theorem guarantees that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that $\ln \Upsilon$ is increasing in technology T, decreasing in any task-specific wage w_k, and increasing in any task-specific arrival of price quotes λ_k.

8.2 Multiple Countries

Consider generalizing the argument above to a world of many countries, trading intermediates and final goods with each other. Now Υ is an $\mathcal{N} \times 1$ vector satisfying

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\frac{\phi}{\theta}} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k},$$

for $n = 1, ..., \mathcal{N}$.

Let $\ln \Upsilon$ be the corresponding vector with $\ln \Upsilon_n$ in place of Υ_n for $n = 1, ..., \mathcal{N}$. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the mapping F, whose n'th element is:

$$\begin{aligned} F_n(y) &= \ln \left[\sum_i \exp \left(\ln \left(T_i d_{ni}^{-\theta} \right) + \sum_k \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right] \\ & \ln \left[\sum_i \exp \left(A_{ni} + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right], \end{aligned}$$

where

$$A_{ni} = \ln\left(T_i d_{ni}^{-\theta}\right) - \sum_{k\in\Omega^0} \theta\beta_k \ln w_{k,i}$$

and

$$u_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}.$$

We can check Blackwell's conditions again. For monotonicity, it is readily apparent that for a vector $x \leq y$ we have $F_n(x) \leq F_n(y)$ for all $n = 1, ..., \mathcal{N}$. For discounting, consider $a > 0$ so that

$$\begin{aligned}
F_n(y+a) &= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}(y_i+a)} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \left[\frac{\phi}{\theta}a + \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right]\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right)\right] \\
&\leq \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] + \beta^P a \\
&= F_n(y) + \beta^P a.
\end{aligned}$$

Thus, even with multiple countries, we can still compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with an $\mathcal{N} \times 1$ vector $y^{(0)}$ (which could simply be a vector of zeros). This method is justified, since the contraction mapping theorem guarantees (just as in the scalar case) that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that each element of $\ln \Upsilon$ is increasing in technology anywhere T_i, decreasing in any task-specific wage $w_{k,i}$ in any country, and increasing in any task-specific arrival of price quotes $\lambda_{k,i}$ in any country. An important caveat, however, is that these comparative statics take task-specific wages as given, so do not predict general-equilibrium outcomes.

Table 1: Customers per French Exporter

	Destination Market			
	Lithuania	Denmark	UK	Germany
Market Size ($billions)	18	94	882	1480
Customers per Exporter:				
Mean	1.5	2.6	5.8	9.6
Percentiles:				
25th	1	1	1	1
50th	1	1	2	2
75th	2	2	5	7
90th	3	5	12	18
95th	4	8	21	33
99th	7	19	65	117

Data are for 2005.

Table 2: Baseline Parameter Settings for Simulation

Parameter	symbol	value
Pareto parameters:		
efficiency distribution	theta	4.0
supply congestion	gamma	0.5
demand congestion	funnyphi	0.2
Technology level per person	T_i/L_i	3.6
World labor force	L	1.0
Labor by type (fractions of labor force):	L^l	
nonproduction (service)		0.4
production		0.6
Iceberg trade cost	d_ni	1.2
Bilateral lambda	lambda_ni	0.25
Tasks, by type:		
service tasks:		
number of tasks	K	4
total share	beta	0.4
production tasks:		
number of tasks	K	24
total share	beta	0.6
Task shares in consumption (same as for production)	alpha	
Expected number of subtasks per task	E_k	1.0
Outsourcing parameters:	lambda^k	
service		0.0
production		1.0

Table 3: Aggregate Results of Simulation

	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Production value added:						
Share of GDP	0.128	0.133	0.133	0.131	0.130	0.129
Share of gross production	0.17	0.20	0.20	0.19	0.18	0.17
Fraction of production tasks outsourced	0.72	0.67	0.67	0.69	0.70	0.71
Gross production per worker	0.63	0.54	0.60	0.68	0.74	0.79
Import share of production	0.98	0.87	0.47	0.31	0.23	0.18
Wage:						
service	1.46	1.34	1.48	1.64	1.75	1.84
production	0.41	0.44	0.49	0.51	0.52	0.53
Skill premium (service/production)	3.58	3.05	3.04	3.22	3.36	3.48
Real wage:						
service	4.16	3.60	3.59	3.79	3.93	4.06
production	1.16	1.18	1.18	1.17	1.17	1.17
Welfare (real per capita consumption)	2.36	2.15	2.14	2.22	2.27	2.32

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Table 4: Firm-Level Results of Simulation

	Country Size					
	L=0.001	L=0.009	L=0.09	L=0.2	L=0.3	L=0.4
Measures of firms:						
producing	0.01	0.03	0.36	0.80	1.15	1.47
selling	0.05	0.21	0.91	1.40	1.72	1.98
Measures normalized by Labor:						
producing	4.5	3.8	4.0	4.0	3.8	3.7
selling	46.0	23.4	10.1	7.0	5.7	4.9
Fraction of firms exporting only	0.85	0.48	0.07	0.03	0.02	0.02
Source country is "France" (L=0.3):						
Mean # customers per firm	1.67	2.45	4.64	5.80	14.48	6.83
Size distribution (percentiles):						
25th	1	1	1	1	1	1
50th	1	1	1	2	2	2
75th	2	2	4	4	8	5
90th	3	4	9	11	26	13
95th	4	7	16	20	54	24
99th	10	19	48	65	199	80

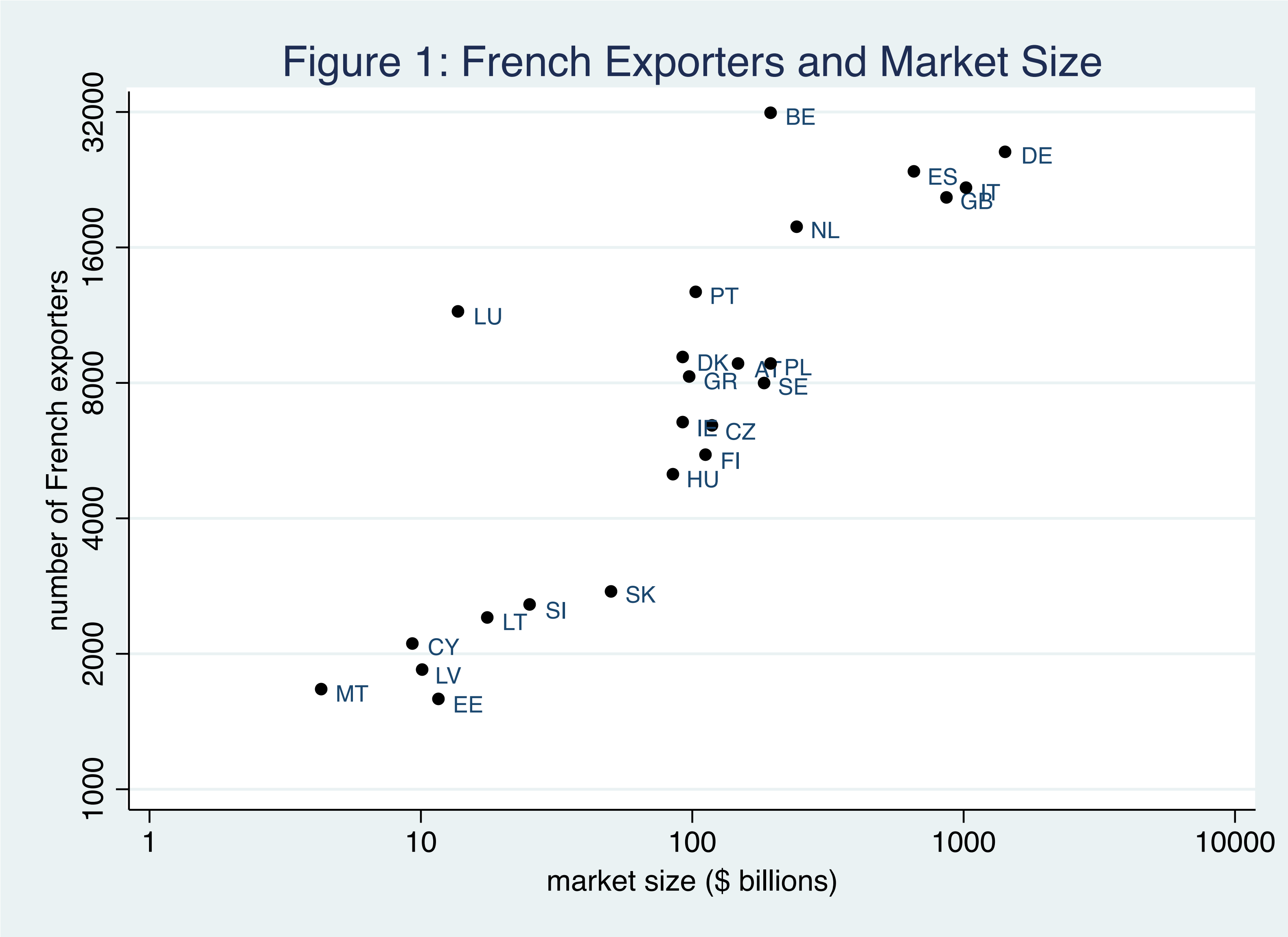


Figure 1: French Exporters and Market Size
number of French exporters
1000
2000
4000
8000
16000
32000
market size ($ billions)
1
10
100
1000
10000
BE
DE
ES
IT
GB
NL
PT
LU
DK
AT
PL
GR
SE
IE
CZ
FI
HU
SK
SI
LT
CY
LV
MT
EE

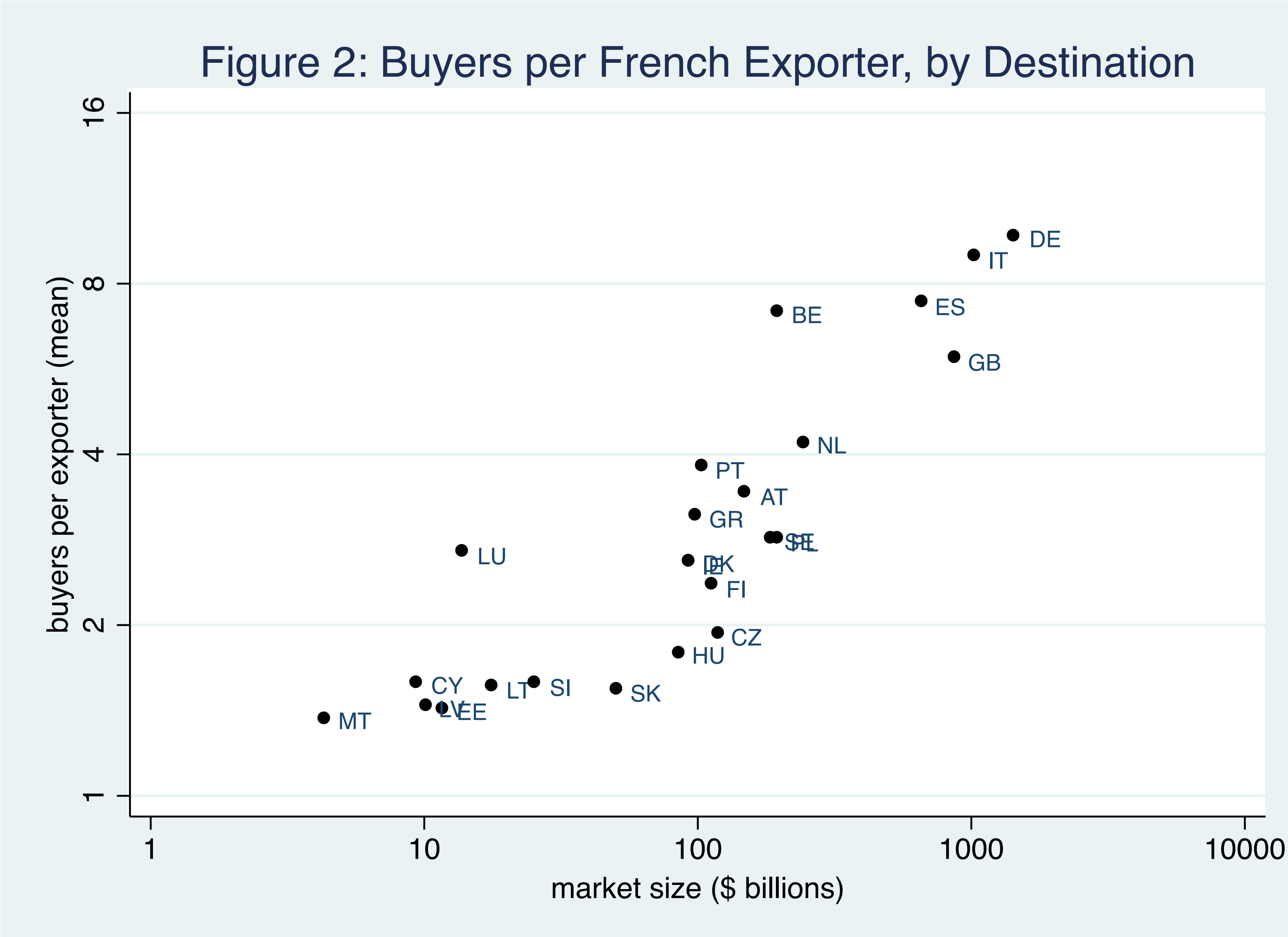


Figure 2: Buyers per French Exporter, by Destination
buyers per exporter (mean)
16
8
4
2
1
market size ($ billions)
1
10
100
1000
10000
DE
IT
ES
BE
GB
NL
PT
AT
GR
LU
FI
CZ
HU
CY
LT
SI
SK
EE
MT

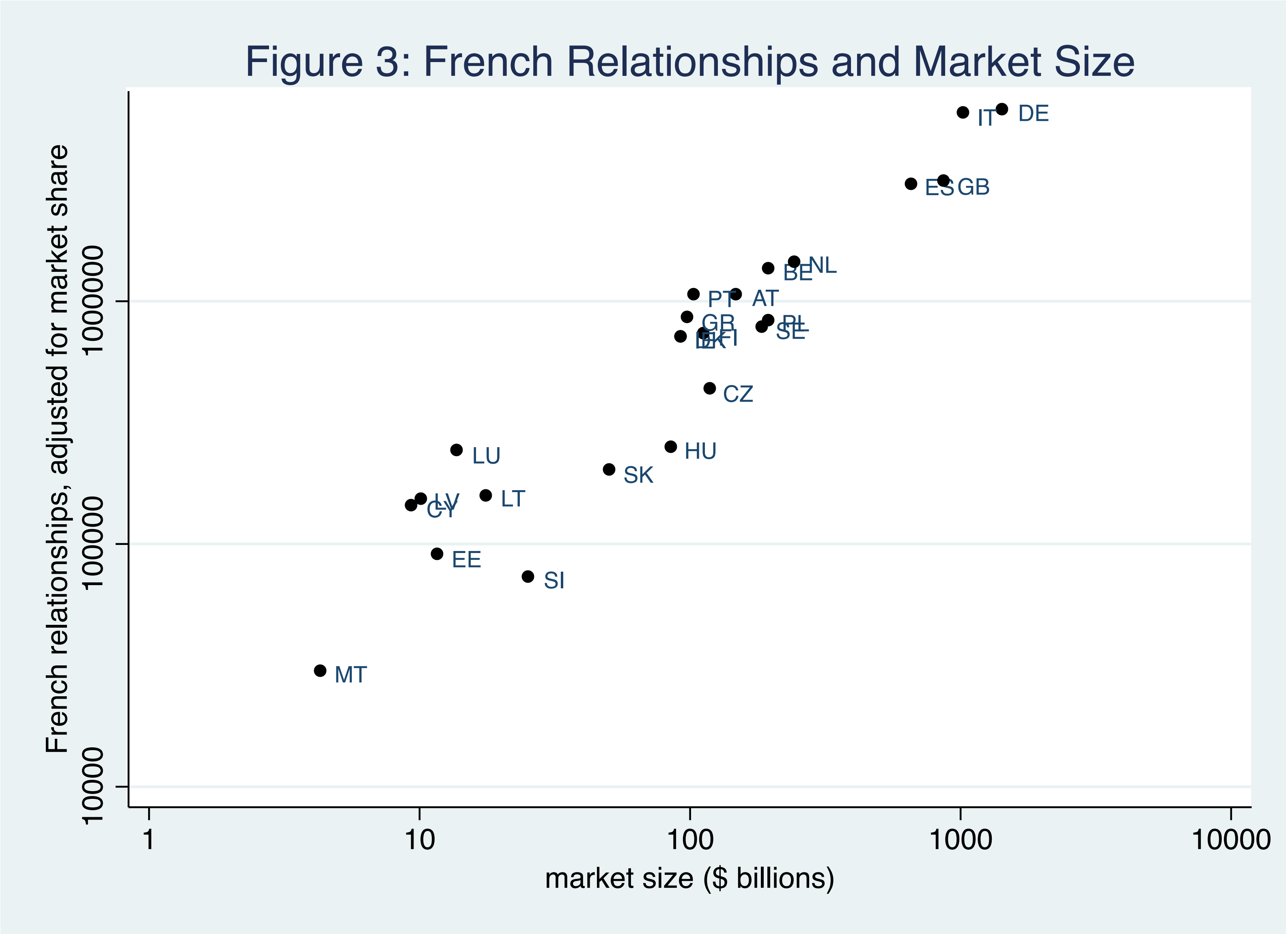
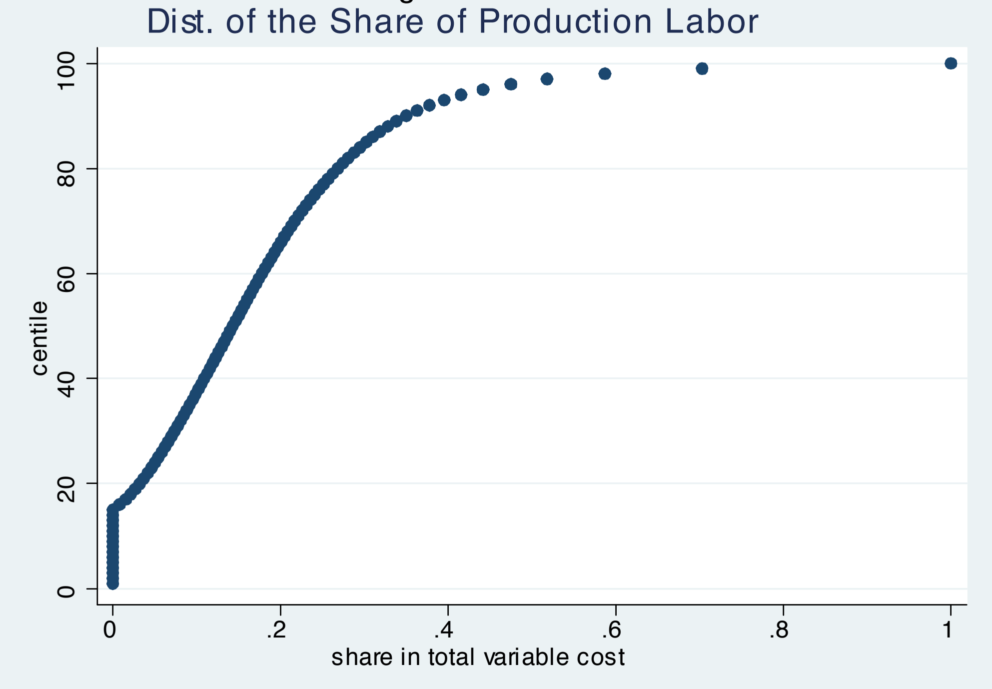


Figure 3: French Relationships and Market Size
French relationships, adjusted for market share
10000
100000
1000000
market size ($ billions)
1
10
100
1000
10000
IT
DE
ES
GB
NL
BE
PT
AT
GB
PL
SE
DK
FI
CZ
HU
SK
LU
LV
CY
LT
EE
SI
MT

Figure 4


Dist. of the Share of Production Labor
centile
0
20
40
60
80
100
0
.2
.4
.6
.8
1
share in total variable cost